

# **LECTURE NOTE ON GEOTECHNICAL ENGINEERING-I**

**5<sup>TH</sup> SEMESTER  
B.TECH (Civil Engineering)**



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Mrs. Sasmita sahu

## **Module-I**

Introduction: Origin of soils, formation of soils, clay mineralogy and soil structure, basic terminology and their relations, index properties of soils. Soil classification: Particle size distribution, use of particle size distribution curve, Particle size classification, textural classification, HRB classification, Unified classification system, Indian standard soil classification system, Field identification of soils. capillary tension, capillary siphoning. Stress conditions in soil: Total stress, pore pressure and effective stress

## **Module-II**

Permeability: Darcy's law, permeability, factors affecting permeability, determination of permeability (laboratory and field methods), permeability of stratified soil deposits. Estimation of yield from wells.

Seepage analysis: Seepage pressure, quick condition, Laplace equation for two –dimensional flow, flow net, properties and methods of construction of flow net, application of flow net, seepage through anisotropic soil and non-homogenous soil, seepage through earth dam. Inverted filter and design of inverted filter.

## **Module-III**

Soil compaction: Compaction mechanism, factors affecting compaction, effect of compaction on soil properties, density moisture content relationship in compaction test, standard and modified proctor compaction tests, field compaction methods, relative compaction, compaction control.

## **Module-IV**

Shear strength of soils: Mohr's stress circle, theory of failure for soils, determination of shear strength (direct shear test, tri-axial compression test, unconfined compression test, van shear test), shear characteristics of cohesionless soils and cohesive soils.

## **Module-V**

Soil consolidation: Introduction, spring analogy, one dimensional consolidation, Terzaghi's theory of one dimensional consolidation, consolidation test, determination of coefficient of consolidation

## **Reference Books:**

1. Geotechnical Engineering, T.N. Ramamurthy & T.G. Sitharam, S. Chand & Co.
2. Soil Mechanics, T.W. Lambe & Whitman, Wiley Eastern Ltd, Nw Delhi.

## 1.0 Introduction and Phase Relation

The term "**soil**" can have different meanings, depending upon the field in which it is considered. To a geologist, it is the material in the relative thin zone of the Earth's surface within which roots occur, and which are formed as the products of past surface processes. The rest of the crust is grouped under the term "rock".

To an engineer, it is a material that can be:

- **built on:** foundations of buildings, bridges
- **built in:** basements, culverts, tunnels
- **built with:** embankments, roads, dams
- **supported:** retaining walls

**Soil Mechanics** is a discipline of Civil Engineering involving the study of soil, its behaviour and application as an engineering material.

Soil Mechanics is the application of laws of mechanics and hydraulics to engineering problems dealing with sediments and other unconsolidated accumulations of solid particles, which are produced by the mechanical and chemical disintegration of rocks, regardless of whether or not they contain an admixture of organic constituents.

Soil consists of a multiphase aggregation of solid particles, water, and air. This fundamental composition gives rise to unique engineering properties, and the description of its mechanical behavior requires some of the most classic principles of engineering mechanics.

Engineers are concerned with soil's mechanical properties: permeability, stiffness, and strength. These depend primarily on the nature of the soil grains, the current stress, the water content and unit weight.

In the Earth's surface, rocks extend up to as much as 20 km depth. The major rock types are categorized as igneous, sedimentary, and metamorphic.

- **Igneous rocks:** formed from crystalline bodies of cooled magma.
- **Sedimentary rocks:** formed from layers of cemented sediments.
- **Metamorphic rocks:** formed by the alteration of existing rocks due to heat from igneous intrusions or pressure due to crustal movement.

Soils are formed from materials that have resulted from the disintegration of rocks by various processes of physical and chemical weathering. The nature and structure of a given soil depends on the processes and conditions that formed it:

- **Breakdown** of parent rock: weathering, decomposition, erosion.
- **Transportation** to site of final deposition: gravity, flowing water, ice, wind.
- **Environment** of final deposition: flood plain, river terrace, glacial moraine, lacustrine or marine.
- **Subsequent conditions** of loading and drainage: little or no surcharge, heavy surcharge due to ice or overlying deposits, change from saline to freshwater, leaching, contamination.

All soils originate, directly or indirectly, from different rock types.



**1.1 Physical weathering** reduces the size of the parent rock material, without any change in the original composition of the parent rock. Physical or mechanical processes taking place on the earth's surface include the actions of water, frost, temperature changes, wind and ice. They cause disintegration and the products are mainly coarse soils.

The main processes involved are exfoliation, unloading, erosion, freezing, and thawing. The principal cause is climatic change. In exfoliation, the outer shell separates from the main rock. Heavy rain and wind cause erosion of the rock surface. Adverse temperature changes produce fragments due to different thermal coefficients of rock minerals. The effect is more for freeze-thaw cycles.

### **1.2 Chemical weathering**

Chemical weather not only breaks up the material into smaller particles but alters the nature of the original parent rock itself. The main processes responsible are hydration, oxidation, and carbonation. New compounds are formed due to the chemical alterations.

Rain water that comes in contact with the rock surface reacts to form hydrated oxides, carbonates and sulphates. If there is a volume increase, the disintegration continues. Due to leaching, water-soluble materials are washed away and rocks lose their cementing properties.

Chemical weathering occurs in wet and warm conditions and consists of degradation by decomposition and/or alteration. The results of chemical weathering are generally fine soils with altered mineral grains.

The effects of weathering and transportation mainly determine the basic *nature* of the soil (size, shape, composition and distribution of the particles).

The environment into which deposition takes place, and the subsequent geological events that take place there, determine the *state* of the soil (density, moisture content) and the *structure* or fabric of the soil (bedding, stratification, occurrence of joints or fissures)

Transportation agencies can be combinations of gravity, flowing water or air, and moving ice. In water or air, the grains become sub-rounded or rounded, and the grain sizes get sorted so as to form poorly-graded deposits. In moving ice, grinding and crushing occur, size distribution becomes wider forming well-graded deposits.

In running water, soil can be transported in the form of suspended particles, or by rolling and sliding along the bottom. Coarser particles settle when a decrease in velocity occurs, whereas finer particles are deposited further downstream. In still water, horizontal layers of successive sediments are formed, which may change with time, even seasonally or daily.

Wind can erode, transport and deposit fine-grained soils. Wind-blown soil is generally uniformly-graded. A glacier moves slowly but scours the bedrock surface over which it passes. Gravity transports materials along slopes without causing much alteration.

### **1.3 BASICS OF SOIL CLASSIFICATION**

Soils as they are found in different regions can be classified into two broad categories:

**(1) Residual soils**

**(2) Transported soils**

### 1.3.1 Residual Soils

Residual soils are found at the same location where they have been formed. Generally, the depth of residual soils varies from 5 to 20 m.

Chemical weathering rate is greater in warm, humid regions than in cold, dry regions causing a faster breakdown of rocks. Accumulation of residual soils takes place as the rate of rock decomposition exceeds the rate of erosion or transportation of the weathered material. In humid regions, the presence of surface vegetation reduces the possibility of soil transportation.

As leaching action due to percolating surface water decreases with depth, there is a corresponding decrease in the degree of chemical weathering from the ground surface downwards. This results in a gradual reduction of residual soil formation with depth, until unaltered rock is found.

Residual soils comprise of a wide range of particle sizes, shapes and composition.

### 1.3.2 Transported Soils

Weathered rock materials can be moved from their original site to new locations by one or more of the transportation agencies to form transported soils. Transported soils are classified based on the mode of transportation and the final deposition environment.

(a) Soils that are carried and deposited by rivers are called ***alluvial deposits***.

(b) Soils that are deposited by flowing water or surface runoff while entering a lake are called ***lacustrine deposits***. Alternate layers are formed in different seasons depending on flow rate.

(c) If the deposits are made by rivers in sea water, they are called ***marine deposits***. Marine deposits contain both particulate material brought from the shore as well as organic remnants of marine life forms.

(d) Melting of a glacier causes the deposition of all the materials scoured by it leading to formation of ***glacial deposits***.

(e) Soil particles carried by wind and subsequently deposited are known as ***aeolian deposits***.

Soil is not a coherent solid material like steel and concrete, but is a particulate material. Soils, as they exist in nature, consist of solid particles (mineral grains, rock fragments) with water and air in the voids between the particles. The water and air contents are readily changed by changes in ambient conditions and location.

As the relative proportions of the three phases vary in any soil deposit, it is useful to consider a soil model which will represent these phases distinctly and properly quantify the amount of each phase. A schematic diagram of the three-phase system is shown in terms of weight and volume symbols respectively for soil solids, water, and air. The weight of air can be neglected.

## 1.4 PHASE RELATIONSHIP OF SOILS

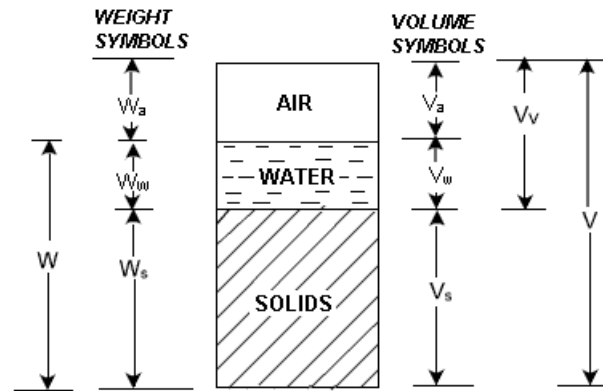


Fig.1: Three phase diagram of soil mass

The soil model is given dimensional values for the solid, water and air components.

Total volume,  $V = V_s + V_w + V_v$

Soils can be partially saturated (with both air and water present), or be fully saturated (no air content) or be perfectly dry (no water content).

In a saturated soil or a dry soil, the three-phase system thus reduces to two phases only, as shown.

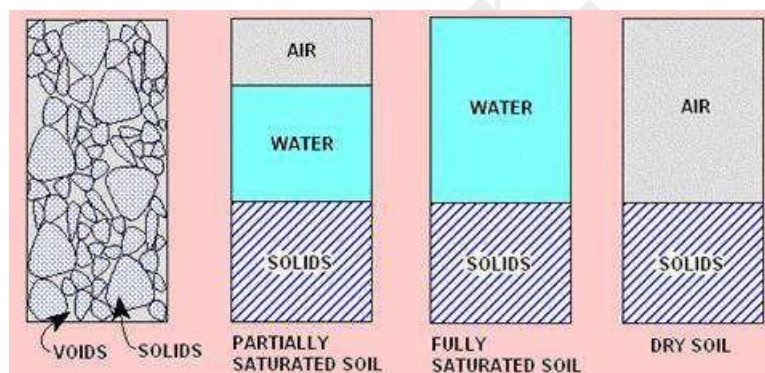


Fig.2: Soil mass showing three phase

For the purpose of engineering analysis and design, it is necessary to express relations between the weights and the volumes of the three phases.

The various relations can be grouped into:

- **Volume relations**
- **Weight relations**
- **Inter-relations**
- As the amounts of both water and air are variable, the volume of solids is taken as the reference quantity. Thus, several relational volumetric quantities may be defined. The following are the **basic volume relations**:

1. Void ratio ( $e$ ) is the ratio of the volume of voids ( $V_v$ ) to the volume of soil solids ( $V_s$ ), and is expressed as a decimal.

$$e = \frac{V_v}{V_s}$$

Eq.1

2. Porosity ( $n$ ) is the ratio of the volume of voids to the total volume of soil ( $V$ ), and is expressed as a percentage.

$$n = \frac{V_v}{V} \times 100 \quad \text{Eq.2}$$

Void ratio and porosity are inter-related to each other as follows:

$$e = \frac{n}{1-n}$$

$$n = \frac{e}{1+e}$$

And

3. The volume of water ( $V_w$ ) in a soil can vary between zero (i.e. a dry soil) and the volume of voids. This can be expressed as the degree of saturation ( $S$ ) in percentage.

$$S = \frac{V_w}{V_v} \times 100 \quad \text{Eq.3}$$

For a dry soil,  $S = 0\%$ , and for a fully saturated soil,  $S = 100\%$ .

4. Air content ( $a_c$ ) is the ratio of the volume of air ( $V_a$ ) to the volume of voids.
- $$a_c = \frac{V_a}{V_v} \times 100 \quad \text{Eq.4}$$

5. Percentage air voids ( $n_a$ ) is the ratio of the volume of air to the total volume.

$$n_a = \frac{V_a}{V} \times 100 = n \times a_c \quad \text{Eq.5}$$

Density is a measure of the quantity of mass in a unit volume of material. Unit weight is a measure of the weight of a unit volume of material. Both can be used interchangeably. The units of density are  $\text{ton/m}^3$ ,  $\text{kg/m}^3$  or  $\text{g/cm}^3$ . The following are the **basic weight relations**:

1. The ratio of the mass of water present to the mass of solid particles is called the **water content ( $w$ )**, or sometimes the **moisture content**.

$$\omega = \frac{W_w}{W_s} \times 100 \quad \text{Eq.6}$$

Its value is 0% for dry soil and its magnitude can exceed 100%.

2. The mass of solid particles is usually expressed in terms of their **particle unit weight ( $\gamma_s$ )** or **specific gravity ( $G_s$ )** of the soil grain solids.

$$\gamma_s = \frac{W_s}{V_s} = G_s \gamma_w \quad \text{Eq.7}$$

where,  $\gamma_w$  = unit weight of water

For most inorganic soils, the value of  $G_s$  lies between 2.60 and 2.80. The presence of organic material reduces the value of  $G_s$ .

3. Dry unit weight ( $\gamma_d$ ) is a measure of the amount of solid particles per unit volume.

$$\gamma_s = \frac{W_s}{V} \times 100 \quad \text{Eq.8}$$

4. Bulk unit weight ( **$\gamma$  or  $\gamma_t$** ) is a measure of the amount of solid particles plus water per unit volume.

$$\gamma_t = \gamma = \frac{W_s + W_w}{V_s + V_w} \times 100 \quad \text{Eq.9}$$

5. Saturated unit weight ( $\gamma_{sat}$ ) is equal to the bulk density when the total voids is filled up with water.

6. Buoyant unit weight or submerged unit weight ( $\gamma'$ ) is the effective mass per unit volume when the soil is submerged below standing water or below the ground water table.

$$\gamma' = \gamma_{sat} - \gamma_w \text{-----} \quad \text{Eq.10}$$

It is important to quantify the state of a soil immediately after receiving in the laboratory and prior to commencing other tests. The water content and unit weight are particularly important, since they may change during transportation and storage.

Some physical state properties are calculated following the practical measurement of others. For example, dry unit weight can be determined from bulk unit weight and water content. The following are some **inter-relations**:

### 1.5 Relationships Involving Porosity, Void Ratio, Degree of Saturation, Water Content, Percent Air Voids and Air Content

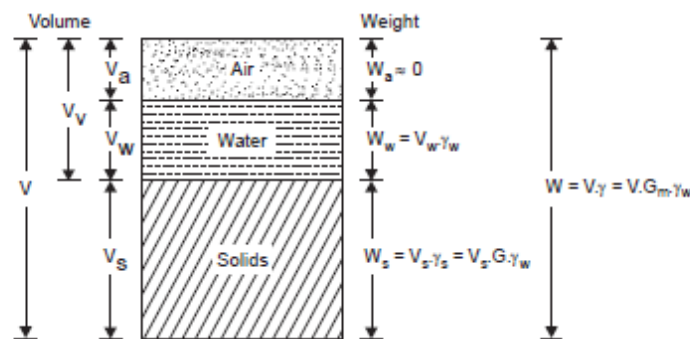


Fig.3: Three phase diagram used for establishment of inter relationship

$$n = \frac{V_v}{V}, \text{ as a fraction}$$

$$= \frac{V - V_s}{V} = 1 - \frac{V_s}{V} = 1 - \frac{W_s}{G \gamma_w V} \text{-----}$$

$$n = 1 - \frac{W_d}{G \gamma_w V} \text{-----} \quad \text{Eq.12}$$

This may provide a practical approach to determine  $n$

$$e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s} = \frac{V}{V_s} - 1 = \frac{V G \gamma_w}{W_s} - 1 \text{-----} \quad \text{Eq.13}$$

$$e = \frac{V G \gamma_w}{W_d} - 1 \text{-----} \quad \text{Eq.14}$$

The expression provides an approach to determine value of void ratio  $e$

$$n = \frac{V_v}{V} \text{ and } e = \frac{V_v}{V_s}$$

$$\frac{1}{n} = \frac{V}{V_v} = \frac{V_s + V_v}{V_v} = \frac{V_s}{V_v} + \frac{V_v}{V_v} = \frac{1}{e} + 1 = \frac{1+e}{e} \text{-----} \quad \text{Eq.15}$$

$$\text{So } n = \frac{e}{1+e} \text{-----} \quad \text{Eq.16}$$

$$a_v = \frac{V_a}{V_v} \text{ and } n = \frac{V_v}{V}$$

$$\therefore \quad na_c = \frac{V_a}{V} = n_a \text{-----} \quad \text{Eq.17}$$

By definition

$$\omega = \frac{W_w}{W_s} \text{-----} \quad \text{Eq.18}$$

$$S = \frac{V_w}{V_v} \text{-----} \quad \text{Eq.19}$$

$$\text{And } e = \frac{V_v}{V_s} \text{-----} \quad \text{Eq.20}$$

$$\text{Now, } Se = \frac{V_w}{V_s} \text{-----} \quad \text{Eq.21}$$

$$\text{Again, } \omega = \frac{W_w}{W_s} = \frac{V_w \gamma_w}{V_s \gamma_s} = \frac{V_w \gamma_w}{V_s G \gamma_w} = \frac{V_w}{V_s G} = Se/G \text{-----} \quad \text{Eq.22}$$

$$\text{Hence } \omega G = Se \text{-----} \quad \text{Eq.23}$$

### 1.6 Relationships Involving Unit Weights, Grain Specific Gravity, Void Ratio, and Degree of Saturation:

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_v} = \frac{W_s(1 + W_w/W_s)}{V_s(1 + V_v/V_s)} \text{-----} \quad \text{Eq.24}$$

$$\text{But } \frac{W_w}{W_s} = \omega, \text{ water content and void ratio } e = \frac{V_v}{V_s}$$

$$\text{Again } \gamma_s = \frac{W_s}{V_s} = G \gamma_w \text{-----} \quad \text{Eq.25}$$

$$\text{Hence, } \gamma = \frac{G \gamma_w}{1 + e} \text{-----} \quad \text{Eq.26}$$

$$\text{Or } \gamma = \gamma_w \frac{(G + G\omega)}{(1 + e)} \text{-----} \quad \text{Eq.27}$$

$$\text{We know that, } \omega G = eS \text{-----} \quad \text{Eq.28}$$

Substituting, we get

$$\gamma = \frac{(G + es)}{(1 + e)} \gamma_w \text{ (where } S \text{ is taken as a fraction)-----} \quad \text{Eq.29}$$

This is a general expression from which unit weight corresponding to the saturated and dry states of soil mass may be found out by substituting  $S=1$  and  $S=0$  respectively.

$$\therefore \gamma_{sat} = \frac{(G + e)}{(1 + e)} \gamma_w \text{-----} \quad \text{Eq.1.30}$$

$$\text{And } \gamma = \frac{G}{(1 + e)} \gamma_w \text{-----} \quad \text{Eq.1.31}$$

The submerged unit weight may be written as

$$\gamma' = \gamma_{sat} - \gamma_w \text{-----} \quad \text{Eq.1.32}$$

$$= \frac{(G + e)}{(1 + e)} \gamma_w - \gamma_w \text{-----} \quad \text{Eq.1.33}$$

$$= \gamma_w \left[ \frac{(G + e)}{(1 + e)} - 1 \right] \text{-----} \quad \text{Eq.1.34}$$

$$\text{So } \gamma' = \frac{(G - 1)}{(1 + e)} \gamma_w \text{-----} \quad \text{Eq.1.35}$$



$$\frac{w}{(1+e)}$$

**Example 1:** A soil has void ratio = 0.72, moisture content = 12% and  $G_s = 2.72$ . Determine its  
 (a) Dry unit weight

(b) Moist unit weight, and the

(c) Amount of water to be added per  $\text{m}^3$  to make it saturated.

**Solution:**

$$\text{a) } \gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{2.72 \times 9.81}{1+0.72} = 15.51 \text{ kN/m}^3$$

b)

$$\gamma = \gamma_d(1 + \omega)$$

$$= \frac{1+0.12}{1+0.72} \times 2.12 \times 9.81 = 17.28 \text{ kN/m}^3$$

$$\text{c) } \gamma_{sat} = \frac{G_s + eS}{1+e} \times \gamma_w = \frac{2.72+0.72}{1+0.72} \times 9.81 = 19.62 \text{ kN/m}^3$$

- Water to be added per  $\text{m}^3$  to make the soil saturated
- $\gamma_{sat} - \gamma$

$$= 19.62 - 17.38 = 2.24 \text{ kN}$$

**Example 2:** The dry density of sand with porosity of 0.387 is  $1600 \text{ kg/m}^3$ . Find the void ratio of the soil and the specific gravity of the soil solids. [Take  $\gamma_w = 1000 \frac{\text{kg}}{\text{m}^3}$ ]

$$n = 0.387$$

$$\gamma_d = 1600 \text{ kg/m}^3$$

**Solution:**

$$e = \frac{n}{1-n} = \frac{0.387}{1-0.387} = 0.631$$

$$G_s = \frac{1+e}{\gamma_w} \times \gamma_d = \frac{1+0.631}{1000} \times 1600 = 2.61$$

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

### Example 3

A sample of saturated soil has a water content of 35%. The specific gravity of soil solids is 2.65. Determine its voids ratio, porosity, saturated unit weight and dry unit weight.

**Saturated soil**

Water content,	$w = 35\%$
specific gravity of solids,	$G = 2.65$
Void ratio,	$e = wG$ , in this case.
$\therefore$	$e = 0.35 \times 2.65 = 0.93$

Porosity,

$$n = \frac{e}{1+e} = \frac{0.93}{1.93} = 0.482 = 48.20\%$$

Saturated unit weight,

$$\begin{aligned}\gamma_{Sat} &= \frac{(G+e)}{(1+e)} \cdot \gamma_w \\ &= \frac{(2.65+0.93)}{(1+0.93)} \times 9.81 \\ &= 18.15 \text{ kN/m}^3\end{aligned}$$

Dry unit weight,

$$\begin{aligned}\gamma_d &= \frac{G \cdot \gamma_w}{(1+e)} \\ &= \frac{2.65 \times 9.81}{1.93} \\ &= 13.44 \text{ kN/m}^3.\end{aligned}$$

**Example 4**

A sample of clay taken from a natural stratum was found to be partially saturated and when tested in the laboratory gave the following results. Specific gravity of soil particle = 2.6, wet weight of sample = 2.50 N, dry weight of sample = 2.10 N and volume of sample = 150 cm<sup>3</sup>. Determine the degree of saturation.

**Sol:**

Data given

Specific gravity of soil particles,	$G = 2.60$
Wet weight,	$W = 2.50 \text{ N};$
Volume,	$V = 150 \text{ cm}^3$
Dry weight,	$W_d = 2.10 \text{ N}$

Water content,  $w = \frac{(W - W_d)}{W_d} \times 100 = \frac{(2.5 - 2.1)}{2.1} \times 100\%$   
 $= \frac{0.40}{2.10} \times 100\% = 19.05\%$

Bulk unit weight,  $\gamma = W/V = 2.50/150 = 0.0167 \text{ N/cm}^3$   
 $= 16.38 \text{ kN/m}^3$

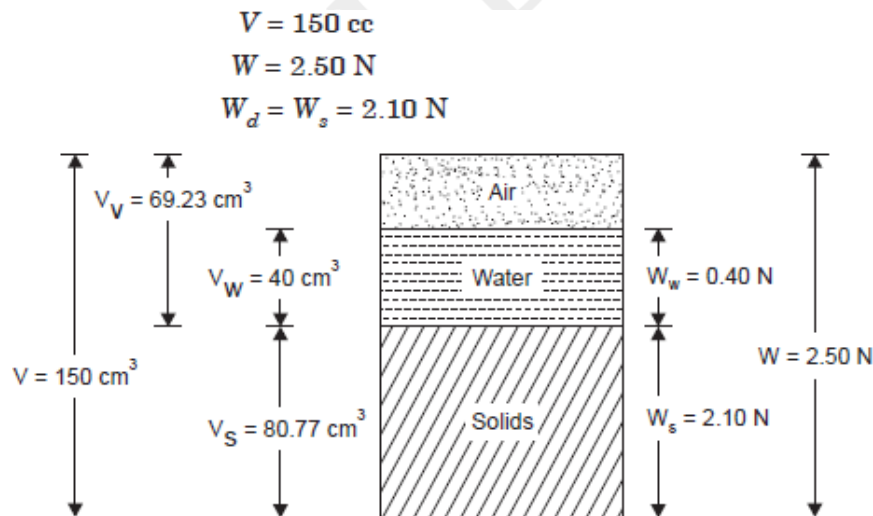
Dry unit weight,  $\gamma_d = \frac{\gamma}{(1+w)} = \frac{16.38}{(1+0.1905)} \text{ kN/m}^3$   
 $= 13.76 \text{ kN/m}^3$

[Also,  $\gamma_d = \frac{W_d}{V} = 2.10/150 = 0.014 \text{ N/cm}^3 = 13.734 \text{ kN/m}^3$ ]

But  $\gamma_d = \frac{G \cdot \gamma_w}{(1+e)}$   
 $13.76 = \frac{2.6 \times 9.81}{(1+e)}$   
 $(1+e) = \frac{2.6 \times 9.81}{13.76} = 1.854$   
 $e = 0.854$

Degree of saturation,  $S = \frac{wG}{e} = \frac{0.1905 \times 2.6}{0.854} = 0.58$   
 $= 58\%$

Alternatively, this can be found from three phase diagram



$$W_w = (2.50 - 2.10) \text{ N}$$

$$= 0.40 \text{ N}$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{0.40}{0.01} = 40 \text{ cm}^3,$$

$$V_s = \frac{W_s}{\gamma_s} = \frac{W_s}{G \cdot \gamma_w} = \frac{2.10}{2.6 \times 0.01} = 80.77 \text{ cm}^3$$

$$V_v = (V - V_s) = (150 - 80.77) = 69.23 \text{ cm}^3$$

$$\text{Degree of saturation, } S = \frac{V_w}{V_v}$$

$$S = 40/69.23 = 0.578$$

$$\therefore S = 40/69.23 = 0.578$$

$$\therefore \text{Degree of saturation} = 57.8\%$$

**Problem No 1** A soil sample has a porosity of 40 %. The specific gravity of soil solids is 2.70. Calculate a) Voids ratio, b) dry density, c) unit weight if the soil is 56% saturated and d) unit weight if the soil is completely saturated

## 2.0 INDEX PROPERTIES OF SOILS

The properties of soil which are used for identification and classification are called as Index Properties of Soil

### INTRODUCTION

Basically in soil mechanics properties of soil are broadly classified as

- 1 Index Properties and
- 2 Engineering Properties

The properties of soil which are used for identification and classification are called as Index Properties of Soil.

### 2.1 PROPERTIES OF SOIL

Divided into Two Categories namely

#### 1 Index Properties

- a. Specific Gravity
- b. Water Content
- c. Particle Size

Distribution

- d. In-Situ Density
- e. Consistency

Limits

- f. Relative Density

## 2 Engineering Properties

- a. Permeability
- b. Shear Strength
- c. Compressibility

### 2.1.1 SPECIFIC GRAVITY

#### Definition:

The ratio of weight of soil solids to the weight of equal volume of water content at 4°C standard temperature

#### Laboratory Determination:

- a. Pycnometer Method

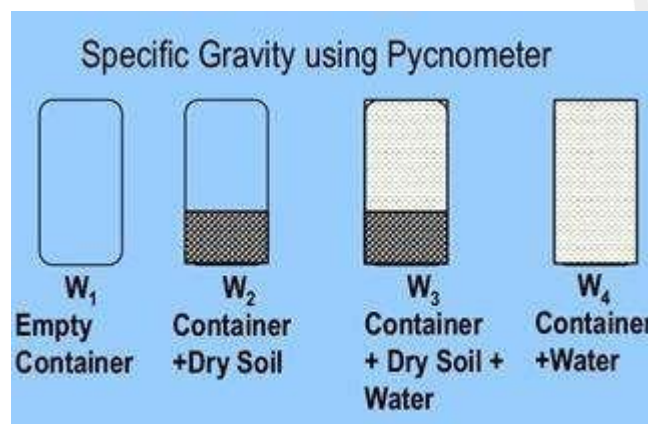


Fig.4: Schematic diagram for determination of Specific gravity

$$\text{Specific gravity } G = \frac{(W_2 - W_1)}{(W_2 - W_1) - (W_3 - W_4)} \text{-----} \quad \text{Eq.2.1}$$

### 2.1.2 DETERMINATION WATER CONTENT

1. Oven Dry Method
2. Sand Bath Method
3. Pycnometer Method
4. Rapid Moisture Meter

#### 1 OVEN DRY METHOD

1. Take the empty weight of container ( $W_1$ )
2. Take the weight of container along with moist soil sample ( $W_2$ )
3. Place the container in the oven for 24 hours at 105°C temperature ( $W_3$ )
4. Note down the weight of container along with dried soil sample ( $W_4$ )

$$\text{Water content } w = \frac{(W_2 - W_3)}{(W_4 - W_1)} \text{-----} \quad \text{Eq.2.2}$$

## 2 SAND BATH METHOD

1. It is a rapid (Quick) and approximate method
2. Sand is collected in a container and is literally heated with fire
3. Take the empty weight of container (W1)
4. Take the weight of container along with moist soil sample (W2)
5. Place the container in the sand bath for 15 to 20 minutes. (W3)
6. Note down the weight of container along with dried soil sample.(w4)

$$\text{Water content } w = \frac{(W_2 - W_3)}{(W_4 - W_1)}$$

## 3 RAPID MOISTURE METER METHOD



It is also called as Calcium Carbide Method

This test is based on gas pressure developed by the reaction of calcium carbide with moisture of soil.

Wet soil is placed in the cup

Holding the moisture meter horizontally, cup and chamber is brought together.

The meter is shaken for about 15 to 20 seconds.

Calcium carbide reacts with available moisture and releases acetylene gas.

From the calibrated scale of the pressure gauge the percentage of water on total mass of wet soil is obtained.

### 2.1.3 PARTICLE SIZE DISTRIBUTION

#### Coarse Soils (Size > 0.75mm)

- |              |              |
|--------------|--------------|
| a. Dry Sieve | b. Wet Sieve |
| Analysis     | Analysis     |

#### Fine Soils (Size < 75mm)

- |                  |                        |
|------------------|------------------------|
| a. Sedimentation | b. Hydrometer analysis |
|------------------|------------------------|



IS Sieves

**SIEVE ANALYSIS**

Objective of sieve analysis is to classify the soil and to determine gradation.

A set of IS Sieves are taken

A suitable amount of pulverized dry soil (500g) is taken and sieved through different sieves with the help of sieve shaker for 10 – 15 minutes.

Weight of soil retained on each sieve is determined.

Percentage weight of soil retained on each sieve is determined.

Percentage finer is determined.

Plot of Percentage finer v/s Size of the sieve on semi log sheet will give the type of gradation, Effective size, Coefficient of uniformity and Coefficient of curvature,

**TABULATION FOR SIEVE ANALYSIS**

IS Sieve Size	Weight of Soil Retained(gm)	Percentage Retained	Cumulative Percentage	Percentage Finer (100% - Cumulative Retained)
4.75mm				
2mm				
1mm				
600 $\mu$				
425 $\mu$				
212 $\mu$				
150 $\mu$				
75 $\mu$				
Pan				



### 2.1.4 TYPICAL PARTICLE SIZE DISTRIBUTION GRAPH

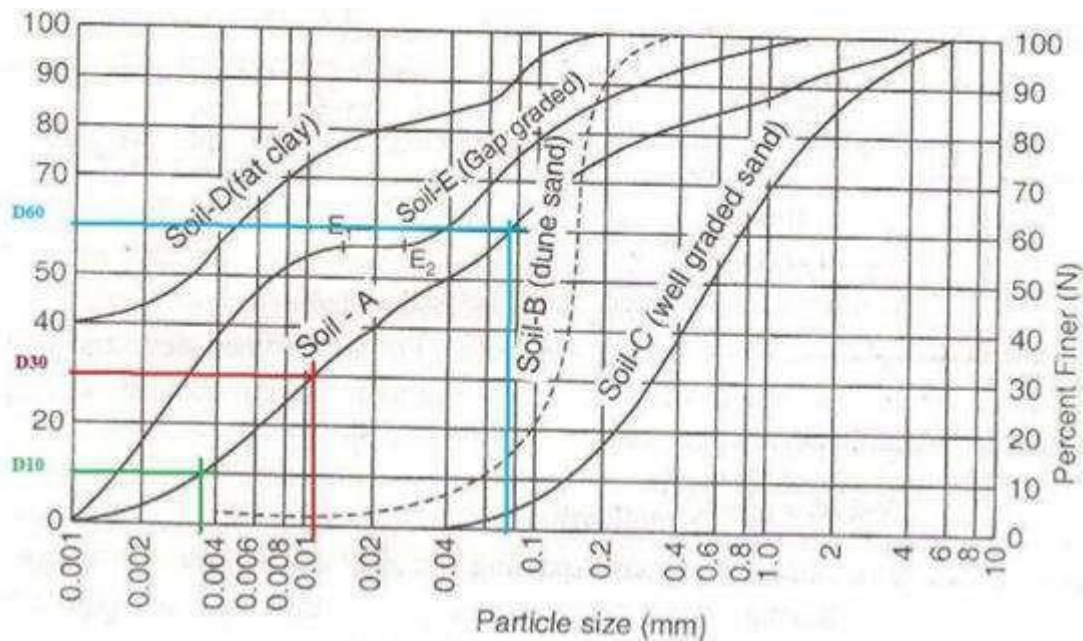


Fig. 5: Grain size distribution curve

#### Gradation and Coefficients

1. Well Graded
2. Uniformly Graded
3. Gap Graded or Step Graded

##### 1. Coefficient of Uniformity ( $C_u$ )

$$C_u = \frac{D_{60}}{D_{10}} \text{-----} \quad \text{Eq.2.3}$$

##### 2. Coefficient of Curvature ( $C_c$ )

$$C_c = \frac{D_{30}^2}{D_{60} \times D_{10}} \text{-----} \quad \text{Eq.2.4}$$

Well graded soils have high  $C_u$  values and poorly graded soils have low  $C_u$  values. If all the particles of soil mass are of the same size  $C_u$  is unity.

$C_c$  lies between 1 to 3 for well graded soil.

Uniform soil:  $C_u = 1$

Poorly graded soil:  $1 < C_u < 4$

Well graded soil:  $C_u > 4$

### 2.1.5 HYDROMETER ANALYSIS

1. The analysis is carried out in two stages

- Calibration of hydrometer
- Sedimentation analysis

#### CALIBRATION OF HYDROMETER

A = Area of stem or graduate

$R_h$  = Hydrometer Reading

$H_e$  = Effective Height

$H$  = Depth of stem up to hydrometer reading

$V_h$  = Volume of hydrometer in cc

$$H_e = \left( H + \frac{h}{2} + \frac{V_h}{2A} \right) - \frac{V_h}{A}$$

Eq.2.5

Hydrometer is immersed in the soil suspension, hydrometer readings are noted at different time intervals

Effective height values are calculated for corresponding 'H' values.

A plot of  $H_e$  v/s  $R_h$  is used as the calibration chart.

Note that calibration of hydrometer is done only once. The same chart can be used for any 'n' number of trials.

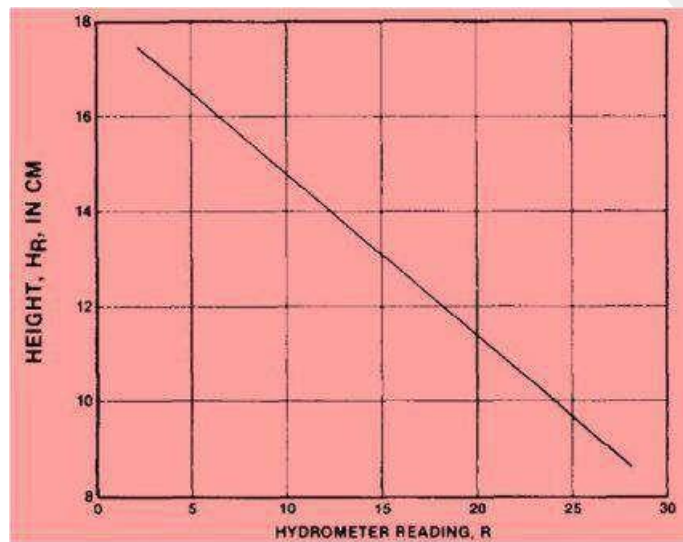


Fig. 6: Calibration chart for Hydrometer

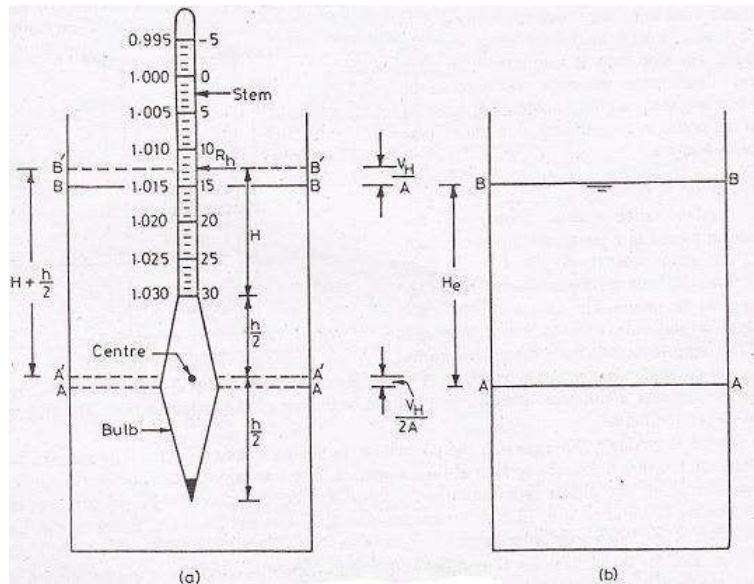


Fig.7: Calibrated Hydrometer

### TEST PROCEDURE

1. 100 ml Soil suspension is prepared by mixing about 12 to 30 gm of soil with dispersing agent such as **sodium hexameta-phosphate (33 gm)** and **sodium carbonate (7 gm)** and **distilled water**
2. 100 ml of soil suspension is added with 900 ml of water and is poured in jar.
3. Suspension is mixed thoroughly and the hydrometer is immersed in the jar.
4. Hydrometer readings are noted at different interval of time.
5.  $H_e$  is taken from the calibration chart
6. Diameter of the particles is determined by following equation

$$D = \sqrt{\frac{18\eta}{(\gamma_s - \gamma_w)}} \times \frac{H_e}{t} \text{-----} \quad \text{Eq.2.6}$$

D - Diameter of soil particle

H - Viscosity of water

$H_e$  - Effective height

t - Time in (seconds)

$\gamma_s$  - Unit weight of solids

$\gamma_w$  - Unit weight of water

% finer is obtained from the following equation

$$N = \left( \frac{G}{G-1} \right) \frac{R_c}{W_s} \times 100 \text{-----} \quad \text{Eq.2.7}$$

Where,

N - % Finer

G - Specific Gravity of Solids

Rc - Corrected reading of hydrometer

Ws -Weight of solids

### *CORRECTIONS FOR HYDROMETER READING*

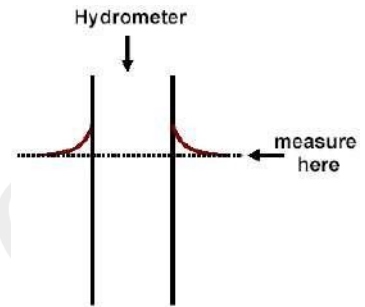
#### *1. Correction for meniscus (cm)*

$$cm = m_u - m_l$$

Where,  $m_u$  = Upper meniscus

$m_l$  = Lower meniscus

Cm is always positive because upper meniscus shows lesser values than actual



#### *2. Correction for temperature (ct)*

More the temperature than standard temperature, (27°C), less will be viscosity, hydrometer reading will be less than actual and hence correction will be positive

Lesser the temperature than standard temperature, (27 °C), more will be viscosity, hydrometer reading will be more than actual and hence correction will be negative.

#### *3. Correction for dispersing agent (cd)*

Addition of dispersing agent increases net density of the sample.

This causes more values than the actual.

Hence, correction for dispersion agent will always be negative.

Total correction (cr)

$$Rc = cm \pm ct - cd$$

### *2.1.6 INSITU DENSITY BY CORE CUTTER METHOD*

Used for cohesive soil in which core can be easily penetrated

Take empty weight of core (Without collar) (W1)

Penetrate the cone into the earth using rammer

After chipping off extra soil, note down weight of core along with soil (W2)

Determine moisture content of the soil using oven dry method.

Bulk unit weight and dry unit weight can be determined as follows

Measure the volume of core (V)

$$\text{In-situ Density} = \gamma = \frac{(W_2 - W_1)}{V} \text{-----} \quad \text{Eq.2.8}$$

$$\text{Dry Density } \rho_d = \frac{\gamma}{1+w} \text{-----} \quad \text{Eq.2.9}$$

### 2.1.7 INSITU DENSITY BY SAND REPLACEMENT METHOD

This test is done in two steps

A. Calibration of container

B. Determination of in-situ density of soil

Sand is filled in the Sand Pouring Cylinder (SPC) and the weight is taken with closed shutter. (W1)

The SPC is placed on a smooth glass plate and the sand is allowed to run out of cone.

Weight of SPC after running sand in to cone is noted (W2)

SPC is refilled and placed on calibrating container and sand is allowed to run out.

The weight of SPC is noted after running sand in to calibrating container (W3).

Weight of sand in cone

$$W_{\text{cone}} = W_1 - W_2$$

Weight of sand in calibrating container

$$W_{\text{cc}} = W_1 - W_3 + W_{\text{cone}}$$

Volume of calibrating container is determined ( $V_{\text{cc}}$ )

$$\text{Unit weight of sand } \gamma_{\text{sand}} = \frac{W_{\text{cc}}}{V_{\text{cc}}} \text{-----} \quad \text{Eq.2.10}$$

SPC with sand up to W1 is taken to the field where bulk unit weight is required. The surface is cleaned and leveled.

The tray with central hole is placed in position. A hole of approximately same size as calibrating container is dug

The removed soil is collected on the sides of tray and its weight is noted ( $W_{\text{soil}}$ ).

The SPC is placed on the hole and sand is allowed to run in to hole.

Weight of SPC is noted (W4).

$$\text{Weight of sand in the hole } W_5 = W_1 - W_4 + W_{\text{cone}}$$

Volume of the hole

$$V_{\text{soil}} = \frac{W_5}{\gamma_{\text{soil}}} \text{-----} \quad \text{Eq.2.11}$$

$$\text{In-situ Density of Soil } \gamma = \frac{W_{\text{soil}}}{V_{\text{soil}}} \text{-----} \quad \text{Eq.2.12}$$



## 2.3 ATTERBERG LIMITS

Consistency is the relative ease with which soil can be deformed.

It is applicable to fine grained soils whose consistency depends on water content. Relative consistency can be expressed as very stiff, stiff, medium stiff, soft, very soft with increasing water content.

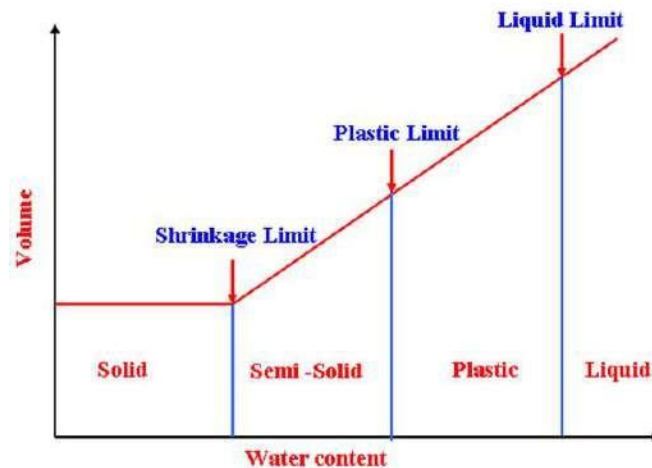


Fig.8: Chart showing consistency limit

### 2.3.1 Liquid Limit

It is the arbitrary water content between plastic and liquid phase of soil

It is the minimum water content in the soil where it behaves as a plastic material

It is the amount of water in the soil, where the soil contains least amount of shear strength.

### 2.3.2 Plastic Limit

It is the minimum water content in the soil at which a thread formed by rolling on a glass plate crumbles at 3 mm diameter.

In other words, it is the arbitrary water content between plastic and semi solid state.

### 2.3.3 Shrinkage Limit

It is the maximum water content at which soil behaves as a solid material.

It is the maximum water content below which, decrease in water content results in no change of volume of soil.

In other words, it is the arbitrary water content between solid and semi solid phase of soil.

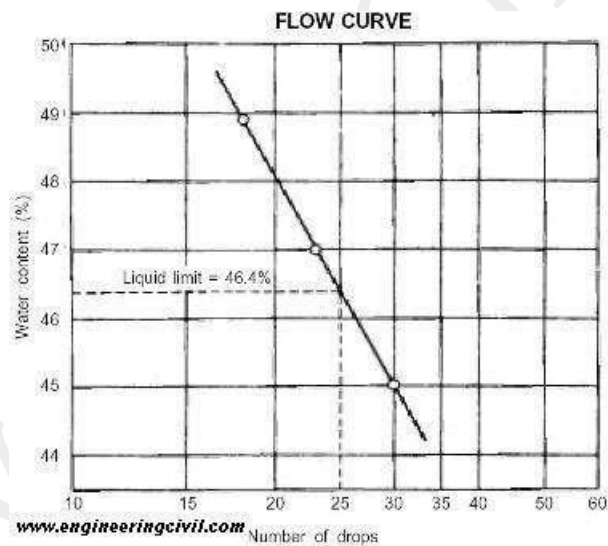
## 2.4 DETERMINATION OF LIQUID LIMIT

*Casagrande's Liquid Limit Apparatus*

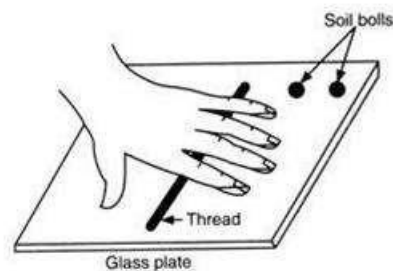
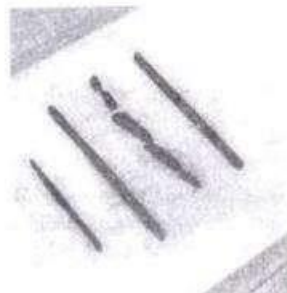
*The following procedures are used*

- 1 Take 120 g of air dried soil after passing through 425 $\mu$  sieve.
- 2 Arbitrarily mix certain amount of water and make a paste.
- 3 Put the soil pat in the brass cup and make a V shaped groove using grooving tool.
- 4 Give blows at a speed of 2 rps., till the groove closes.
- 5 Count the number of blows at the end of experiment.
- 6 Take certain amount of soil from the cup to determine water content.
- 7 Take at least 5 number of trials with varying number of blows from 15 to 35

The corresponding **water content at 25th blow is taken as liquid limit**



## 2.5 DETERMINATION OF PLASTIC LIMIT



### Procedures to be taken

- 1 Take about 15 g of air dried soil after passing through 425 $\mu$  sieve.
- 2 Arbitrarily mix certain amount of water and make soil ball.
- 3 Roll the soil ball on glass plate with palm.
- 4 Make thread till it attains a thickness of 3mm.



- 5 If the thread crumbles at 3 mm thick, then take sample to determine water content.
- 6 Corresponding water content is taken as **PLASTIC LIMIT**.

## 2.6 DETERMINATION OF SHRINKAGE LIMIT

### *Procedure to determine Shrinkage Limit*

- 1 Take about 30 g of air dried soil after passing through 425 $\mu$  sieve in an evaporating dish.
- 2 Soil is mixed with sufficient quantity of water so that it may flow.
- 3 Fill the soil mix in dish and take the weight of dish.
- 4 The soil pat is allowed to dry in air till the colour of pat changes from dark to light.
- 5 The dish is then placed in oven at 110oc till it attains constant weight.



Apparatus required for Shrinkage Limit test

- 6 The shrinkage dish is weighed with the dry soil and dry weight of soil is determined.
- 7 The volume of dry soil pat is determined by mercury displacement method.

$V_1$  = Volume of container in which plastic phase soil is filled initially.

$W_1$  = Weight of wet soil sample along with container.

$V_2$  = Volume of soil at shrinkage limit.

$W_2$  = Weight of soil sample at shrinkage limit.

$V_s$  = Weight of dry soil pat

$\gamma_w$  = Unit weight of water

Shrinkage limit is nothing but water content at a particular case i.e,  $w_s$

$$w_s = \frac{W_w}{W_s} \times 100$$

Weight of water at shrinkage limit is,

$$W_{ws} = (W_1 - W_2) - (V_1 - V_2)\gamma_w$$



$$\text{Hence, Shrinkage Limit } w_s = \frac{(W_1 - W_2) - (V_1 - V_2)\gamma_w}{W_s} \quad \text{Eq.2.13}$$

## 2.7 CONSISTENCY INDICES

### 2.7.1 Plasticity Index ( $I_p$ )

It is the range of water content in soil in during which soil behaves as a **plastic material**

Simply it is the difference in the water content at liquid limit and plastic limit

Highly plastic soils exhibit greater plasticity values

$$\text{Plasticity Index } I_p = (w_l - w_p) \quad \text{Eq.2.14}$$

### 2.7.2 Shrinkage Index ( $I_s$ )

It is the range of water content in soil in during which soil behaves as a **semi solid material**

Simply it is the difference in the water content at plastic limit and shrinkage limit

$$I_s = (w_p - w_s) \quad \text{Eq.2.15}$$

Plasticity Index ( $I_p$ )	Soil Description
0	Non Plastic
<7	Low plastic
7 - 17	Medium plastic
>17	Highly plastic

### 2.7.3 Flow Index

It is the slope of the flow curve obtained between the number of blows and water content in the Casagrande's test for the determination of liquid limit.

Flow index indicates the loss of shear strength of soil upon the increase of water.

Greater the slope angle less is the shear strength of soil.

Flatter slope curve indicates more shear strength in the soil.

$$\text{Flow index } F = \frac{(W_1 - W_2)}{\log_{10} \frac{n_2}{n_1}} \quad \text{Eq.2.16}$$

### 2.7.4 Consistency Index ( $I_c$ )

It is the ratio of the difference between the liquid limit and natural water content of soil to its plasticity index. This is very important to attribute the soil property

$$\text{Consistency Index } I_c = \frac{(w_l - w_n)}{I_p} \quad \text{Eq.2.17}$$

Condition	Consistency Index	Soil's nature
If $w_l = w_n$	$I_c = 0$	Soil is at liquid limit.
If $w_l < w_n$	$I_c = \text{Negative}$	Soil is in liquid state.
If $w_l > w_n$	$I_c = \text{Positive}$	Soil is either in solid or semi solid state

### 2.7.5 Liquidity Index ( $I_L$ )

It is the ratio of the difference between the natural water content of soil and its plastic limit to its plasticity index.

Liquidity index varies from 0 to 1

$$\text{Liquidity Index, } I_L = \frac{(w_n - w_p)}{I_p} \text{-----} \quad \text{Eq.2.18}$$

### 2.7.6 Toughness Index ( $I_T$ )

It is the ratio of plasticity index to the flow index.

This gives an idea of shear strength of soil at plastic limit

$$\text{Toughness Index, } I_T = \frac{I_p}{I_F} \text{-----} \quad \text{Eq.2.19}$$

### 2.7.7 Density Index ( $I_D$ )

It is the ratio of difference between the void ratio of a Cohesionless soil in the loosest state and void ratio in its natural state to the difference between its void ratios in the loosest and densest states.

$$\text{Hence Density Index } I_D = \frac{(e_{max} - e_{nat})}{(e_{max} - e_{min})} \text{-----} \quad \text{Eq.2.20}$$

The nature of soil can be predicted from Density Index value as given in table

Density Index ( $I_D$ )	Soil Description
<15 Very loose	15 - 35 Loose
35 - 65 Medium	65- 85 Dense
>85 Very dense	5/
<15 Very loose	15 - 35 Loose
35 - 65 Medium	65- 85 Dense

### 2.7.8 Shrinkage Ratio (SR)

It is the ratio of a given volume change in a soil, expressed as a percentage of the dry volume, to the corresponding change in water content above the shrinkage limit.

$$SR = \frac{(V_1 - V_2)}{V_d} \times 100 \text{-----} \quad \text{Eq.2.21}$$

Where

$V_1$  = Volume of soil at water content  $w_1$  (%)

$V_2$  = Volume of soil at water content  $w_2$  (%)

$V_d$  = Volume of dry soil mass

At Shrinkage,

$V_2 = V_d$

$w_2 = w_s$

$$\text{Shrinkage Ratio (SR)} = \frac{(V_1 - V_d)}{V_d} \times 100 \text{-----} \quad \text{Eq.2.22}$$

$w_1 - w_s$

## 2.8 MISCELLANEOUS PROPERTIES

### 2.8.1 Sensitivity ( $St$ )

It is defined as the ratio of unconfined compressive strength of an undisturbed soil sample to the unconfined compressive strength of the same soil after remolding at unaltered water content.

Sensitivity	Soil Description
1-4	Normal
4 - 8	Sensitive
8 - 15	Extra sensitive
>15	Quick

### 2.8.2 Activity

It is the ratio of plasticity index of soil to the % finer fraction available in it.

Clayey soils exhibit more activity values.

Activity Classification	Activity Classification
<0.75	Inactive
0.75 – 1.25	Normal
>1.25	Active

**Example 1.0** A sample of saturated soil has a water content of 35%. The specific gravity of solids is 2.65. Determine its void ratio, porosity, saturated unit weight and dry unit weight.

**Saturated soil**

Water content,	$w = 35\%$
specific gravity of solids,	$G = 2.65$
Void ratio,	$e = wG$ , in this case.
$\therefore$	$e = 0.35 \times 2.65 = 0.93$
Porosity,	$n = \frac{e}{1+e} = \frac{0.93}{1+0.93} = 0.482 = 48.20\%$
Saturated unit weight,	$\gamma_{Sat} = \frac{(G+e)}{(1+e)} \cdot \gamma_w$ $= \frac{(2.65+0.93)}{(1+0.93)} \times 9.81$ $= 18.15 \text{ kN/m}^3$
Dry unit weight,	$\gamma_d = \frac{G \cdot \gamma_w}{(1+e)}$ $= \frac{2.65 \times 9.81}{1.93}$ $= 13.44 \text{ kN/m}^3.$

**Example 2.0** A sample of clay taken from a natural stratum was found to be partially saturated and when tested in the laboratory gave the following results. Compute the degree of saturation. Specific gravity of soil particles = 2.6 ; wet weight of sample = 2.50 N; dry weight of sample = 2.10 N ; and volume of sample = 150 cm<sup>3</sup>.

Sol:

Data given

Specific gravity of soil particles,	$G = 2.60$
Wet weight,	$W = 2.50 \text{ N};$
Volume,	$V = 150 \text{ cm}^3$
Dry weight,	$W_d = 2.10 \text{ N}$

Water content,	$w = \frac{(W - W_d)}{W_d} \times 100 = \frac{(2.5 - 2.1)}{2.1} \times 100\%$ $= \frac{0.40}{2.10} \times 100\% = 19.05\%$
Bulk unit weight,	$\gamma = W/V = 2.50/150 = 0.0167 \text{ N/cm}^3$ $= 16.38 \text{ kN/m}^3$
Dry unit weight,	$\gamma_d = \frac{\gamma}{(1+w)} = \frac{16.38}{(1+0.1905)} \text{ kN/m}^3$ $= 13.76 \text{ kN/m}^3$

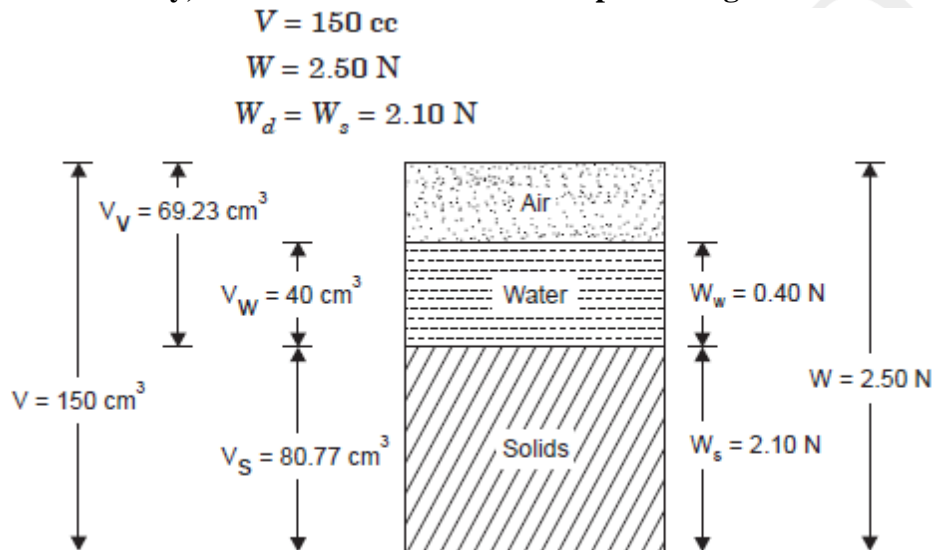
$$\left[ \text{Also, } \gamma_d = \frac{W_d}{V} = 2.10/150 = 0.014 \text{ N/cm}^3 = 13.734 \text{ kN/m}^3 \right]$$

But

$$\begin{aligned} \gamma_d &= \frac{G \cdot \gamma_w}{(1+e)} \\ 13.76 &= \frac{2.6 \times 9.81}{(1+e)} \\ (1+e) &= \frac{2.6 \times 9.81}{13.76} = 1.854 \\ e &= 0.854 \end{aligned}$$

$$\begin{aligned} \text{Degree of saturation, } S &= \frac{wG}{e} = \frac{0.1905 \times 2.6}{0.854} = 0.58 \\ &= 58\% \end{aligned}$$

Alternatively, this can be found from three phase diagram



$$\begin{aligned} W_w &= (2.50 - 2.10) \text{ N} \\ &= 0.40 \text{ N} \end{aligned}$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{0.40}{0.01} = 40 \text{ cm}^3,$$

$$V_s = \frac{W_s}{\gamma_s} = \frac{W_s}{G \cdot \gamma_w} = \frac{2.10}{2.6 \times 0.01} = 80.77 \text{ cm}^3$$

$$V_v = (V - V_s) = (150 - 80.77) = 69.23 \text{ cm}^3$$

$$\text{Degree of saturation, } S = \frac{V_w}{V_v}$$

$$S = 40/69.23 = 0.578$$

$$\therefore S = 40/69.23 = 0.578$$

$$\therefore \text{Degree of saturation} = 57.8\%$$

### 3.0 CLAY MINERALOGY

A soil particle may be a mineral or a rock fragment. A mineral is a chemical compound formed in nature during a geological process, whereas a rock fragment has a combination of one or more minerals. Based on the nature of atoms, minerals are classified as silicates, aluminates, oxides, carbonates and phosphates.

Out of these, silicate minerals are the most important as they influence the properties of clay soils. Different arrangements of atoms in the silicate minerals give rise to different silicate structures.

#### 3.1 Basic Structural Units

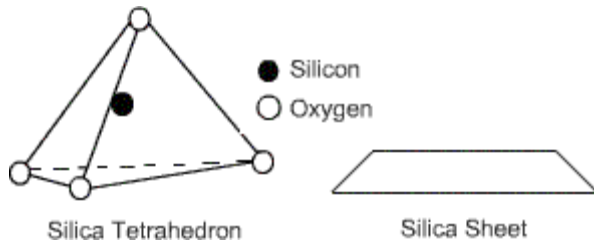
Soil minerals are formed from two basic structural units: tetrahedral and octahedral. Considering the valencies of the atoms forming the units, it is clear that the units are not electrically neutral and as such do not exist as single units.

The basic units combine to form sheets in which the oxygen or hydroxyl ions are shared among adjacent units. Three types of sheets are thus formed, namely *silica sheet*, *gibbsite sheet* and *brucite sheet*.

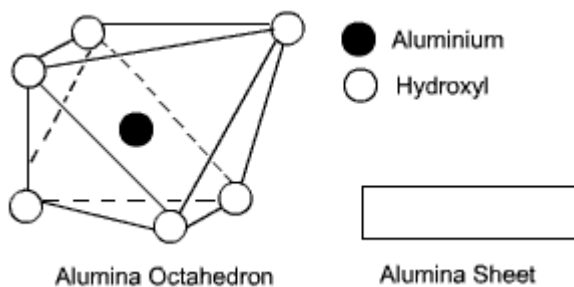
**Isomorphous substitution** is the replacement of the central atom of the tetrahedral or octahedral unit by another atom during the formation of the sheets.

The sheets then combine to form various two-layer or three-layer sheet minerals. As the basic units of clay minerals are sheet-like structures, the particle formed from stacking of the basic units is also plate-like. As a result, the surface area per unit mass becomes very large.

A tetrahedral unit consists of a central silicon atom that is surrounded by four oxygen atoms located at the corners of a tetrahedron. A combination of tetrahedrons forms a *silica sheet*.



An octahedral unit consists of a central ion, either aluminium or magnesium, that is surrounded by six hydroxyl ions located at the corners of an octahedron. A combination of aluminium- hydroxyl octahedrons forms a ***gibbsite sheet***, whereas a combination of magnesium-hydroxyl octahedrons forms a ***brucite sheet***.



Kaolinite and halloysite clay minerals are the most common.

### 3.1.1 Kaolinite Mineral

The basic kaolinite unit is a two-layer unit that is formed by stacking a gibbsite sheet on a silica sheet. These basic units are then stacked one on top of the other to form a lattice of the mineral. The units are held together by hydrogen bonds. The strong bonding does not permit water to enter the lattice. Thus, kaolinite minerals are stable and do not expand under saturation. Kaolinite is the most abundant constituent of residual clay deposits.

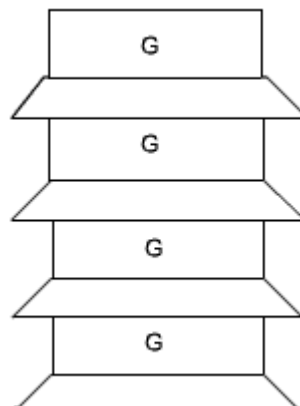


Fig.3.1: Kaolinite Structure

### 3.1.2 Halloysite Mineral

The basic unit is also a two-layer sheet similar to that of kaolinite except for the presence of water between the sheets.

Montmorillonite and illite clay minerals are the most common. A basic three-layer sheet unit is formed by keeping one silica sheet each on the top and at the bottom of a gibbsite sheet. These units are stacked to form a lattice as shown.

### 3.1.3 Montmorillonite Mineral

The bonding between the three-layer units is by vander Waals forces. This bonding is very weak and water can enter easily. Thus, this mineral can imbibe a large quantity of water causing swelling. During dry weather, there will be shrinkage.

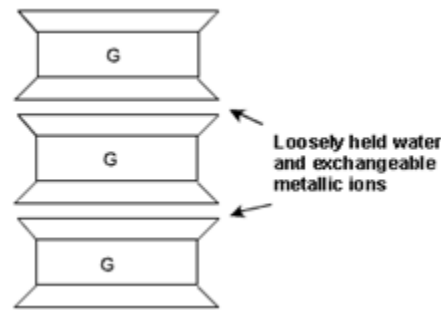


Fig.3.2: Structure of Montmorillonite Mineral

### 3.1.4 Illite Mineral

Illite consists of the basic Montmorillonite units but are bonded by **secondary valence forces** and **potassium ions**, as shown. There is about 20% replacement of aluminium with silicon in the gibbsite sheet due to *isomorphous substitution*. This mineral is very stable and does not swell or shrink.

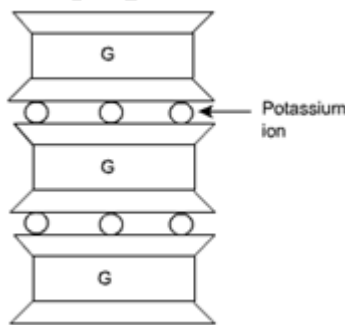


Fig.3.3: Illite Mineral Structure

Natural soils are rarely the same from one point in the ground to another. The content and nature of grains varies, but more importantly, so does the arrangement of these. The arrangement and organisation of particles and other features within a soil mass is termed its **fabric**.

CLAY particles are **flaky**. Their thickness is very small relative to their length & breadth, in some cases as thin as 1/100th of the length. They therefore have high specific surface values. These surfaces carry negative electrical charge, which attracts positive ions present in the pore water. Thus a lot of water may be held as adsorbed water within a clay mass.



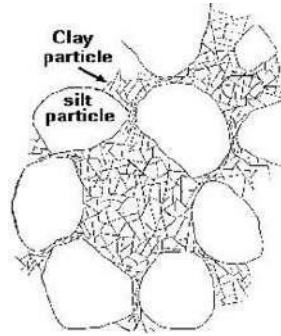


Fig. 3.4: Schematic diagram of Clay Structure

### 3.2 IMPORTANT PROPERTIES OF CLAY MINERALS

Some of the important properties that influence the behaviour of clay minerals are presented below:

#### 3.2.1 High surface area

Specific surface area (SSA) is defined as the surface area of soil particles per unit mass (or volume) of dry soil. Its unit is in  $\text{m}^2/\text{g}$  or  $\text{m}^2/\text{m}^3$ . Clay minerals are characterized by high specific surface area (SSA) as listed in Table 2.2. High specific surface area is associated with high soil- water- contaminant interaction, which indicates high reactivity. The reactivity increases in the order Kaolinite < Illite < Montmorillonite. For the purpose of comparison, SSA of silt and sand has also been added in the table. There is a broad range of SSA values of soils, the maximum being for montmorillonite and minimum for sand. As particle size increases SSA decreases.

For smectite type minerals such as montmorillonite, the primary external surface area amounts to 50 to 120  $\text{m}^2/\text{g}$ . SSA inclusive of both primary and secondary surface area (interlayer surface area exposed due to expanding lattice) and termed as total surface area would be closed to 800  $\text{m}^2/\text{g}$ . For kaolinite type minerals there is possibility of external surface area where in the interlayer surface area does not contribute much.

#### 3.2.2 Plasticity and cohesion

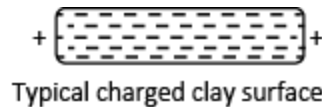
Clay attracts dipolar water towards its surface by adsorption. This induces plasticity in clay. Therefore, plasticity increases with SSA. Water in clays exhibits negative pressure due to which two particles are held close to each other. Due to this, apparent cohesion is developed in clays.

#### Surface charge and adsorption

**Clay surface is charged due to following reasons:**

Iso-morphous substitution (Mitchell and Soga 2005): During the formation of mineral, the normally found cation is replaced by another due to its abundant availability. For example: when  $\text{Al}^{+3}$  replace  $\text{Si}^{+4}$  there is a shortage of one positive charge, which appears as negative charge on

clay surface. Such substitution is therefore the major reason for net negative charge on clay surface.  $O^{2-}$  and  $OH^-$  functional groups at edges and basal surface also induce negative charge. Dissociation of hydroxyl ions or broken bonds at the edges is also responsible for unsatisfied negative or positive charge. Positive charge can occur on the edges of kaolinite plates due to acceptance of  $H^+$  in the acid pH range (Berkowitz et al. 2008). It can be negatively charged under high Ph environment.



Absence of cations from the crystal lattice also contributes to charge formation. In general, clay particle surface are negatively charged and its edges are positively charged.

Due to the surface charge, it would adsorb or attract cations (+ve charged) and dipolar molecules like water towards it. As a result, a layer of adsorbed water exists adjacent to clay surface.

### 3.2.3 Exchangeable cations and cation exchange capacity

Due to negative charge, clay surface attracts cations towards it to make the charge neutral. These cations can be replaced by easily available ions present in the pore solution, and are termed as exchangeable ions. The total quantity of exchangeable cations is termed as cation exchange capacity, expressed in milli equivalents per 100 g of dry clay. Cation exchange capacity (CEC) is defined as the unbalanced negative charge existing on the clay surface. Kaolinite exhibits very low cation exchange capacity (CEC) as compared to montmorillonite. Determination of CEC is done after removing all excess soluble salts from the soil. The adsorbed cations are then replaced by a known cation species and the quantity of known cation required to saturate the exchange sites is determined analytically.

## 3.3 FLOCCULATION AND DISPERSION

When two clay particles come closer to each other it experiences (a) interparticle attraction due to weak van-der-Waal's force (b) repulsion due to -ve charge. When particles are sufficiently close, attraction becomes dominant active force and hence there is an edge to face configuration for clay particles as shown in Fig. 3.5 a. Such a configuration is termed as flocculant structure. When the separation between clay particles increase, repulsion becomes predominant and hence the clay particles follows face to face configuration called dispersed structure. A lot of micro and macro level behaviour of clays are associated with these arrangement of clay particles (Mitchell and Soga 2005).

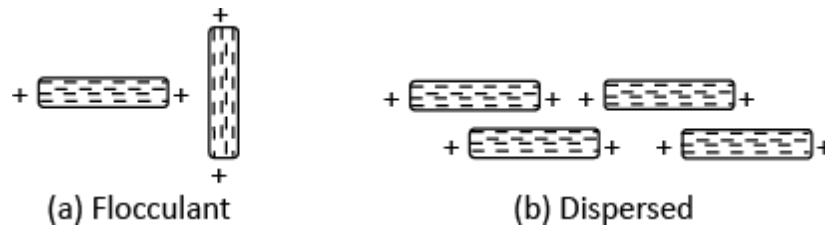


Fig. 3.5: Different arrangement of clay particle

#### 3.4.1 Swelling and shrinkage

Some clay minerals when exposed to moisture are subjected to excessive swelling and during drying undergo excessive shrinkage. A lot of engineering properties of soil is affected by this behaviour and the stability of structures founded on such soils become detrimental. The swelling of clay minerals decreases in the order montmorillonite > illite > kaolinite.

### 3.5 MINERALS OTHER THAN SILICA AND CLAY

Other than silica and clay, subsurface contains a variety of minerals such as oxides and carbonates that governs the reactivity of soil and its interaction with the environment. Some of the abundant metal oxide minerals present are iron oxides (hematite, magnetite, goethite etc.) and aluminium oxides (gibbsite, boehmite). Other oxide minerals (such as manganese oxide, titanium oxide) are far less than Fe and Al oxides, but because of small size and large surface area, they would affect very significantly the geochemical properties of subsurface. These oxides are mostly present in residual soils of tropical regions. Other major components include soluble calcium carbonate and calcium sulphate, which has relatively high surface area. In most soils, quartz is the most abundant mineral, with small amount of feldspar and mica present. Carbonate minerals such as calcite and dolomite are found in some soils in the form of bulky particles, precipitates etc. Sulphate minerals mainly gypsum are found in semiarid and arid regions.

### 3.6 SOIL MINERALOGY CHARACTERIZATION

One of the very well established methods for mineralogy characterization of fine-grained soils is by using X-ray diffraction (XRD) analysis. Majority of the soil minerals are crystalline in nature and their structure is defined by a unique geometry. XRD identifies minerals based on this unique crystal structure. In XRD, characteristic X-rays of particular wave length are passed through a crystallographic specimen. When X-ray interacts with crystalline specimen it gives a particular diffraction pattern, which is unique for a mineral with a particular crystal structure. The diffraction pattern of the soil specimen (according to its crystal structure), which is based on powder diffraction or polycrystalline diffraction, is then analyzed for the qualitative and quantitative (not always) assessment of minerals. Sample preparation method for XRD should be done with great care as the XRD reaches only a small layer (nearly 50  $\mu\text{m}$ ) from the surface of the sample. Hence, homogeneity is very important. Soil sample is initially dried and sieved through 2 mm sieve. Sieved sample is homogenized in a tumbler mixer for 30 min. A control mix of 30 g was taken and ground in lots of 15 g in a gyratory pulverizer. 15 percent by weight

of  $\text{KIO}_4$  (internal standard) was added to 5 g of specimen and again homogenized in a mixer.. X-ray wave of monochromatic radiation ( $\text{K}_\alpha$ ) is commonly obtained from copper radiation, which is commonly known as Cu-  $\text{K}_\alpha$ . A typical XRD output is represented by Fig. 2.5. It can be noted from the figure that ordinate represent relative intensity of X-ray diffraction and abscissa represents twice of angle at which a striking X-ray beam of wave length  $\lambda$  makes with parallel atomic planes. Based on this diffraction pattern, the minerals can be identified by matching the peak with the data provided by International Centre Diffraction Data (ICDD) formerly known as Joint Committee on Powder Diffraction Standards (JCPDS).

It is understood that the area under the peak of diffraction pattern gives the quantity of each phase present in the specimen. However, quantitative determination of mineral composition in soils based on simple comparison of diffraction peak heights or area under peak is complex and uncertain because of different factors such as mineral crystallinity, hydration, surface texture of the specimen, sample preparation, non-homogeneity of soil samples, particle orientation etc. The method of quantification will be more precise for those soils with less number of minerals. Al-Rawas et al. (2001) have discussed about constant mineral standard method and constant clay method for quantification of clay minerals

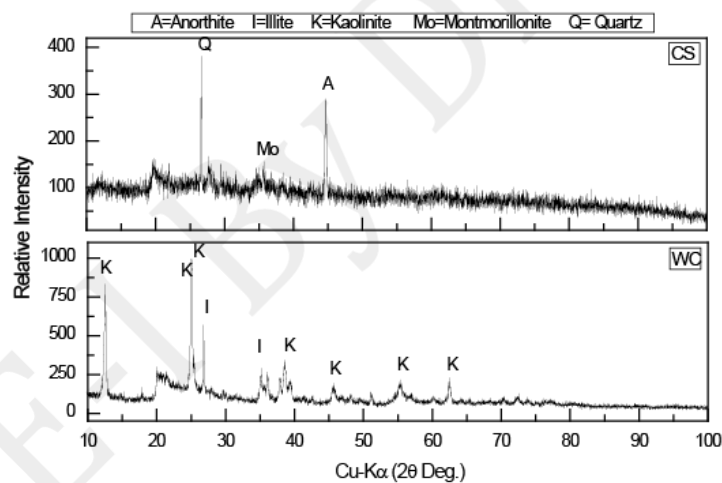


Fig.3.6: A typical XRD pattern with mineral identification for two different soils (modified from Sreedeeep 2006)

In the first method, increasing quantity of clay are added to the fixed mass of known standard and the difference in X-ray diffraction intensity when the specimen changes from 100 percent standard to 100 percent clay is noted. The peak area ratio for each component is then plotted against percentage of clay, based on which regression equation is determined. This regression equation is further used for mineral quantification. In the second method, known weight of pure standard mineral is added to clay containing the same components, and the change in the

reflection peak-area intensity of each component is measured to estimate the weight proportion of that component.

### 3.7 INFLUENCE OF WATER ON CLAY MINERAL BEHAVIOUR

In dry clay, adsorbed cations are bound tightly to the soil surface. The cations in excess, and their associated anions are present as a salt precipitate. When this dry clay is placed in water, the salts go into solution and the adsorbed cations are hydrated. The negatively charge mineral surface and the distributed charge in the adjacent phase are together known as the diffuse (or electrical) double layer. Factors which affect the nature and extent of the double layer include: (i) composition and concentration of the electrolyte solution, (ii) density of charge on the surface of the particle, (iii) valence of the cations, (iv) dielectric constant of the medium, and (v) temperature. Mathematical theories concerning the double layer which include these factors are well summarised in Lee (1968), and Mitchell (1977).

Previously it has been mentioned that clay particles can be of colloid size. In solution, the particles will move in a random manner. If the attraction forces between particles exceed forces of repulsion, the particles will increase in size - up to a stage where gravitational forces will predominate and the particles will sediment out, i.e. the system flocculates. The forces controlling attraction and repulsion can be classified as: (a) Independent of a system: eg. electrostatic or electromagnetic (Van der Waals) or (b) System Dependent: eg. electrostatic (electron or ion clouds) or ion hydration. Electrostatic and electromagnetic forces both contribute to attraction, whilst electron or ion clouds and ion hydration are forces of repulsion, with ion hydration being the major repulsive force.

Thus the behaviour of clay minerals in suspension can be controlled by changing the environment. For the Cardinia Creek reservoir turbidity of the water was reduced by introducing gypsum - the thickness of the double layer of the clay particles in suspension was reduced, leading to forces of attraction exceeding repulsion forces. For this case the enlarged particles settled out. (Grant et al, 1976). Any factor which changes the “thickness” of the double layer will alter the characteristics displayed by a clay soil. Two of the more important system variables are:

(a) Cation Valence: an increase in valence suppresses the concentration of the solution between adjacent particles, and the potential between particles, leading to a decrease in inter-particle repulsion. (b) Ion Size: the larger the ion size the thicker the layer required to accommodate the necessary number of cations, hence the greater the repulsion. Associated with both of these factors is the phenomenon that multi-valent ions are adsorbed preferentially onto the mineral surface. For a given number of exchange sites, the higher the valence the lower the number of ions required to satisfy charge neutrality and hence the double layer is affected accordingly. This feature can be detected through very simple soil tests.

Na Kaolinite

$w_L = 53\%$ ,  $w_p = 32\%$

Ca Kaolinite

$$w_L = 38\%, w_p = 27\%$$

Since the difference between  $w_L$  and  $w_p$  is indicative of the compressibility of the soil we can argue that a Na Kaolinite would be more compressible than a Ca Kaolinite. When the double layer and mineral particle size are considered together it can be argued that the smaller the particle (and hence the higher the specific surface) the larger the amount of water that (with respect to the mass of dry soil) will be associated with the double layer. This feature is of great importance when the swell of a soil, when wetted from a dry condition, is considered. Of particular significance is the clay mineral montmorillonite: the bonding between successive layers is weak and when an ion associated with this interlayer position hydrates, the clay mineral stack will expand. Expansion associated with moisture can cause structural failure in some lightly loaded structures.

### 3.8 STRUCTURE OF SOILS

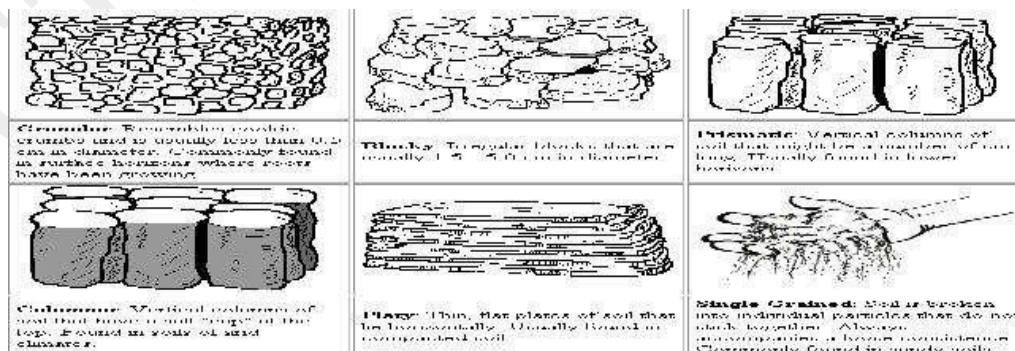
The 'structure' of a soil may be defined as the manner of arrangement and state of aggregation of soil grains. In a broader sense, consideration of mineralogical composition, electrical properties, orientation and shape of soil grains, nature and properties of soil water and the interaction of soil water and soil grains, also may be included in the study of soil structure, which is typical for transported or sediments soils. Structural composition of sedimented soils influences, many of their important engineering properties such as permeability, compressibility and shear strength. Hence, a study of the structure of soils is important.

The following types of structure are commonly studied:

- (a) Single-grained structure
- (b) Honey-comb structure
- (c) Flocculent structure

#### 3.8.1 Single-grained Structure

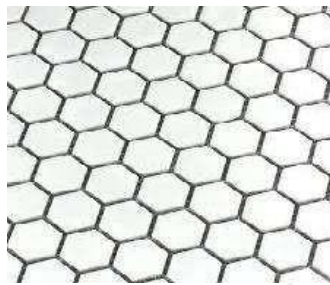
Single-grained structure is characteristic of coarse-grained soils, with a particle size greater than 0.02mm. Gravitational forces predominate the surface forces and hence grain to grain contact results. The deposition may occur in a loose state, with large voids or in a dense state, with less of voids





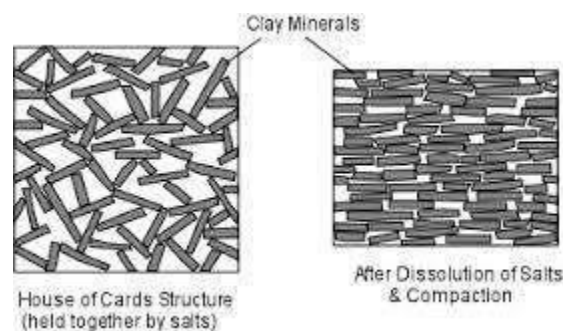
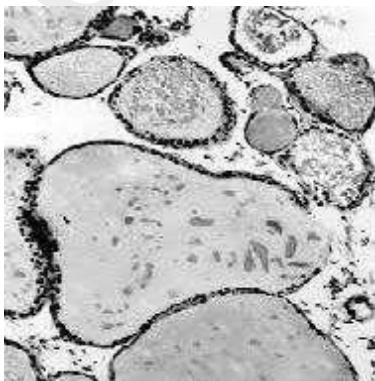
### 3.8.2 Honey-comb Structure

This structure can occur only in fine-grained soils, especially in silt and rock flour. Due to the relatively smaller size of grains, besides gravitational forces, inter-particle surface forces also play an important role in the process of settling down. Miniature arches are formed, which bridge over relatively large void spaces. This results in the formation of a honeycomb structure, each cell of a honey-comb being made up of numerous individual soil grains. The structure has a large void space and may carry high loads without a significant volume change. The structure can be broken down by external disturbances



### 3.8.3 Flocculent Structure

This structure is characteristic of fine-grained soils such as clays. Inter-particle forces play a predominant role in the deposition. Mutual repulsion of the particles may be eliminated by means of an appropriate chemical; this will result in grains coming closer together to form a 'floc'. Formation of flocs is 'flocculation'. But the flocs tend to settle in a honeycomb structure, in which in place of each grain, a floc occurs. Thus, grains grouping around void spaces larger than the grain-size are flocs and flocs grouping around void spaces larger than even the flocs result in the formation of a 'flocculent' structure. Very fine particles or particles of colloidal size ( $< 0.001$  mm) may be in a flocculated or dispersed state. The flaky particles are oriented edge-to-edge or edge-to-face with respect to one another in the case of a flocculated structure. Flaky particles of clay minerals tend to form a card house structure (Lambe, 1953), when flocculated. When inter-particle repulsive forces are brought back into play either by remoulding or by the transportation process, a more parallel arrangement or reorientation of the particles occurs, as shown in Fig. This means more face-to-face contacts occur for the flaky particles when these are in a dispersed state. In practice, mixed structures occur, especially in typical marine soils.



#### 4.0 SOIL CLASSIFICATION SYSTEM

Classification systems are used to group soils according to their order of performance under given set of physical conditions. Soils that are grouped in order of performance for one set of physical conditions will not necessarily have the same order of performance under some other physical conditions.

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##### 4.1 Different Classification of Soils for Engineering Purpose

1. Classification based on grain size
2. Textural classification
3. AASHTO classification system
4. Unified soil classification system

##### 4.1.1 Grain Size Classification System for Soils

Grain size classification systems were based on grain size. In this system the terms clay, silt, sand and gravel are used to indicate only particle size and not to signify nature of soil type. There are several classification systems in use, but commonly used systems are shown here.

	2.00	0.6	0.2	0.06	0.02	0.006	0.002
MIT Gravel	C	M	F	C	M	F	Clay
	Sand			Silt			

##### 4.1.2 Textural Classification of Soil

The classification of soil exclusively based on particle size and their percentage distribution is known as textural classification system. This system specifically names the soil depending on the percentage of sand, silt and clay. The triangular charts are used to classify soil by this system.

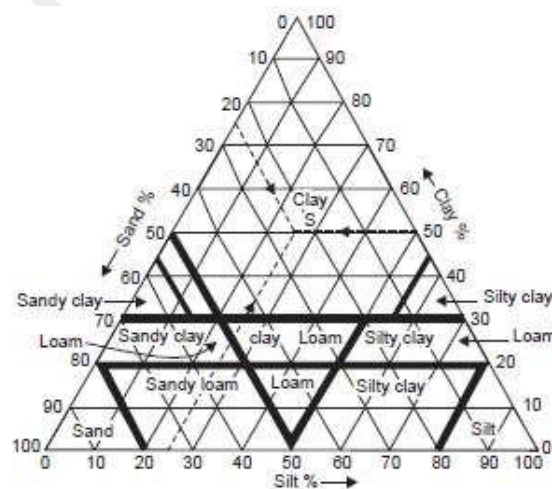


Fig-1: Textural Classification of U.S. Public Roads Administration



### 4.1.3 AASHTO classification system of Soil

	80	20	4.75			0.075	0.002
ISI	C	F	C	M	F	Silt	Clay
	Gravel		Sand				

AASHTO classification is otherwise known as PRA classification system. It was originally developed in 1920 by the U.S. Bureau of Public Roads for the classification of soil for highway sub-grade use.

This system is developed based on particle size and plasticity characteristics of soil mass. After some revision, this system was adopted by the AASHTO in 1945.

In this system the soils are divided into seven major groups. Some of the major groups further divided into subgroups. A soil is classified by proceeding from left to right on the classification chart to find first the group into which the soil test data will fill.

Soil having fine fractions are further classified based on their group index. The group index is defined by the following equation.

$$\text{Group index} = (F - 35)[0.2 + 0.005 (LL - 40)] + 0.01(F - 15)(PI - 10)$$

F – Percentage passing 0.075mm size

LL – Liquid limit

PI – Plasticity index

When the group index value is higher, the quantity of the material is poorer.

General classification	Granular Materials(35% Or less passing 0.075mm sieve)							Silt-clay materials More than 35% passing 0.075mm			
Group classification	A-1		A-3	A-2				A-4	A-5	A-6	A-7-5
	A-1-a	A-1-b		A-2-4	A-2-5	A-2-6	A-2-7				A-7-6
(a) Sieve analysis % passing i) 2.00 mm ii) 0.425 mm iii) 0.075mm	50max 30max 15 max	50max 25max	51min 10max	35max	35max	35max	35max	36min	36min	36min	36min
(b) Characteris tics passing through 0.425 mm i) Liquid limit ii) Plastic limit	6 max		NP	40max 10max	41min 10max	40max 11min	41min 11min	40max 10max	41min 10max	40max 11min	41min 11min*
(c) Usual type of significant constituent materials	Stone Fragments Gravel and Sands		Fine sands	Silty or Clayey Gravel Sands				Silty Soils		Clayey Soils	
(d) General rating as Subgrade	Excellent to Good							Fair to Poor			

Note: \*If Plasticity index is equal to or less than (Liquid limit-30), the soil is A-7-5(PL>30%)

If Plasticity index greater than (Liquid limit-30), the soil is A-7-6(PL<30%)

#### 4.1.4 Unified Soil Classification System

Unified soil classification system was originally developed by Casagrande (1948) and was known as airfield classification system. It was adopted with some modification by the U.S. Bureau of Reclamation and the U.S. Corps of Engineers.

This system is based on both grain size and plasticity characteristics of soil. The same system with minor modification was adopted by ISI for general engineering purpose (IS 1498 – 1970).

IS system divides soil into three major groups, coarse grained, fine grained and organic soils and other miscellaneous soil materials.

Coarse grained soils are those with more than 50% of the material larger than 0.075mm size. Coarse grained soils are further classified into gravels (G) and sands (S). The gravels and sands are further divided into four categories according to gradation, silt or clay content.

Fine grained soils are those for which more than 50% of soil finer than 0.075 mm sieve size. They are divided into three sub-divisions as silt (M), clay (c), and organic salts and clays (O), based on their plasticity nature they are added with L, M and H symbol to indicate low plastic, medium plastic and high plastic respectively.

##### Examples:

GW – well graded gravel, GP – poorly graded gravel, GM – Silty gravel

SW – Well graded sand, SP – Poorly graded sand, SM – Silty sand, SC – Clayey sand

CL – Clay of low plastic, CI – Clay of medium plastic, CH – Clay of higher plastic

ML – Silt of medium plastic, MI – Silt of medium plastic, MH – Silt of higher plastic

OL – Organic silt and clays of low plastic, OI – Organic silt and clays of medium plastic

OH – Organic silt and clays of high plastic.

Fine grained soils have been sub-divided into three subdivisions of low, medium and high compressibility instead of two sub-divisions of the original **Unified Soil Classification System**.

Table-2: Significance of letters for group symbol in table-3.

Soil	Soil Component	Symbol
Coarse Grained	Boulder	None
	Cobble	None
	Gravel	G
	Sand	S
Fine Grained	Silt	M
	Clay	C
	Organic Matter	O
Peat	Peat	Pt
Applicable to Coarse grained Soils	Well graded	W
	Poorly Graded	P
Applicable to Fine grained soils	Low compressibility $W_L < 35$	L
	Medium compressibility ( $W_L$ 35 to 50)	I

The standard recommends that when a soil possesses characteristics of two groups either in particle size distribution or in plasticity, it is designed by combination of group symbols.

Table 3: Unified Soil Classification System (based on particle passing through 75 mm sieve)

Criteria for Assigning Group Symbols				Group Symbol
Coarse-Grained Soils More than 50% of retained on No. 200 sieve	Gravels More than 50% of coarse fraction retained on No. 4 sieve	Clean Gravels	$C_u \geq 4$ and $1 \leq C_c \leq 3^c$	GW
		Less than 5% fines <sup>a</sup>	$C_u < 4$ and/or $1 > C_c > 3^c$	GP
		Gravels with Fines	$PI < 4$ or plots below “A” line (Figure 3.20)	GM
		More than 12% fines <sup>a,d</sup>	$PI > 7$ and plots on or above “A” line (Figure 3.20)	GC
	Sands 50% or more of coarse fraction passes No. 4 sieve	Clean Sands	$C_u \geq 6$ and $1 \leq C_c \leq 3^c$	SW
		Less than 5% fines <sup>b</sup>	$C_u < 6$ and/or $1 > C_c > 3^c$	SP
		Sands with Fines	$PI < 4$ or plots below “A” line (Figure 3.20)	SM
		More than 12% fines <sup>b,d</sup>	$PI > 7$ and plots on or above “A” line (Figure 3.20)	SC
Fine-Grained Soils 50% or more passes No. 200 sieve	Silts and Clays Liquid limit less than 50	Inorganic	$PI > 7$ and plots on or above “A” line (Figure 3.20) <sup>e</sup>	CL
			$PI < 4$ or plots below “A” line (Figure 3.20) <sup>e</sup>	ML
		Organic	$\frac{\text{Liquid limit—oven dried}}{\text{Liquid limit—not dried}} < 0.75$ ; see Figure 3.20; OL zone	OL
	Silts and Clays Liquid limit 50 or more	Inorganic	$PI$ plots on or above “A” line (Figure 3.20)	CH
			$PI$ plots below “A” line (Figure 3.20)	MH
		Organic	$\frac{\text{Liquid limit—oven dried}}{\text{Liquid limit—not dried}} < 0.75$ ; see Figure 3.20; OH zone	OH
Highly Organic Soils	Primarily organic matter, dark in color, and organic odor			Pt

<sup>a</sup> Gravels with 5 to 12% fine require dual symbols: GW-GM, GW-GC, GP-GM, GP-GC.

<sup>b</sup> Sands with 5 to 12% fines require dual symbols: SW-SM, SW-SC, SP-SM, SP-SC.

$$^c C_u = \frac{D_{60}}{D_{10}}; \quad C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}}$$

<sup>d</sup> If  $4 \leq PI \leq 7$  and plots in the hatched area in Figure 3.16, use dual symbol GC-GM or SC-SM.

<sup>e</sup> If  $4 \leq PI \leq 7$  and plots in the hatched area in Figure 3.16, use dual symbol CL-ML.

## 4.2 INDIAN STANDARD CLASSIFICATION SYSTEM FOR SOIL

Indian Standard Classification System (ISC) was adopted by Bureau of Indian Standards is in many respect similar to the Unified Soil Classification (USC) system.

**Soils are divided into three broad divisions:**

1. Coarse grained soils, when 50% or more of the total material by weight is retained on 75 micron IS sieve.
2. For fine grained soils, when more than 50% of the total material passes through 75 micron IS sieve.
3. If the soil is highly organic and contains a large percentage of organic matter and particles of decomposed vegetation, it is kept in a separate category marked as peat (Pt).

In all there are 18 groups of soils: 8 groups of coarse grained, 9 groups of fine grained and one of peat.

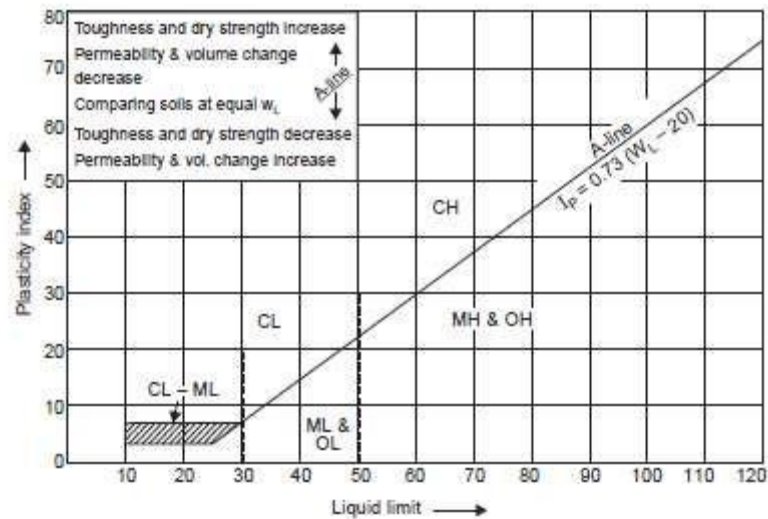


Table 4: Basic Soil Components in ISC System

Soil	Soil components	Symbol	Particle size range and description
Coarse grained	Boulder	None	Rounded to angular, hard, rock, particle average diameter more than 300 mm
	Cobble	None	Rounded to angular, hard, rock, particle average diameter smaller than 300 mm but retained on 80 mm IS sieve.
	Gravel	G	Rounded to angular, hard, rock, particle average diameter smaller than 80 mm but retained on 4.75 mm IS sieve. Coarse : 80 to 20 mm IS sieve Fine: 20 mm to 425 micron IS sieve
	Sand	S	Rounded to angular, hard, rock, particle passing through 4.75 mm but retained on 75 micron IS sieve. Coarse: 4.75 mm to 2 mm IS sieve Medium: 2.00 mm to 425 micron Fine: 425 micron to 65 micron IS sieve
Fine grained	Silt	M	Particle smaller than 75 micron, Identify behavior that slightly plastic or non-plastic regardless of moisture and exhibits little or no strength when air dried.
	Clay	C	Particle smaller than 75 micron, Identify behavior that can exhibit Plastic properties within certain considerable strength when air dried
	Organic matter	O	Organic matter in various sizes and stage of decomposition.

Table: 5 Classification of Coarse-grained Soils (ISC) System

Division	Sub-division		Group Symbol	Typical Names	Laboratory Criteria		Remark
Coarse grained soil (More than half of material is larger than the 75 – micron IS sieve)	Gravel More than half of material is larger than the 4.75 mm IS sieve	Clean gravel (Fine less than 5%)	GW	Well graded gravel	$C_u > 4$ $C_c$ between 1 & 3		When fines are between 5% to 12% boarder line cases requiring use of dual symbol GW-GP.SW-SC etc
			GP	Poor graded gravel	Not meeting all requirement for GW		
		Gravel with fines (Fines more than 12%)	GM	Silty gravel	Atterberg limit below A line or $I_p < 4$	Atterberg limits plotting above A line with $I_p$ between 4 & 7 are boarder line cases requiring use of dual symbol GM-GC	
			GC	Clayey gravel	Atterberg limit above A line or $I_p > 7$		
	Sands More than half of material is smaller than the 4.75 mm IS sieve	Clean sand (Fine less than 5%)	SW	Well graded sands	$C_u > 6$ $C_c$ between 1 & 3		
			SP	Poor graded sands	Not meeting all requirement for SW		
		Sands with fines (Fines more than 12%)	SM	Silty Sands	Atterberg limit below A line or $I_p < 4$	Atterberg limits plotting above A line with $I_p$ between 4 & 7 are boarder line cases requiring use of dual symbol SM-SC	
			SC	Clayey Sands	Atterberg limit above A line or $I_p > 7$		

Table: 6 Classification of Fined-grained Soils (ISC) System

Division	Sub-division	Group Symbol	Typical Names	Laboratory Criteria		Remark
Fine grained soil (More than half of material is smaller than the 75 μ IS sieve )	Low compressibility (L) Liquid limit less than 35%	ML	Organic silts with none to low plasticity	Atterberg limit below A line or $I_p < 7$	Atterberg limits plotting above A line with $I_p$ between 4 & 7 (hatched zone) ML-CL	Black cotton soils in India lies partly above and partly below the A-Line
		CL	Inorganic clays with low plasticity	Atterberg limit above A line or $I_p > 7$		
		OL	Organic silts with low plasticity	Atterberg limit below A line		
	Intermediate compressibility (I) Liquid limit more than 35% but less than 50%	MI	Inorganic silts with medium plasticity	Atterberg limit below A line		
		CI	Inorganic clays with medium plasticity	Atterberg limit above A line		
		OI	Organic silts with medium plasticity	Atterberg limit below A line		
	High compressibility (H) Liquid limit more than 50%	MH	Inorganic silts with High plasticity	Atterberg limit below A line		
		CH	Inorganic clays with High plasticity	Atterberg limit above A line		
		OH	Organic silts with medium to High plasticity	Atterberg limit below A line		

### Field Identification of Soils

In field identification of soil, the engineer concerned first determines whether the soil is coarse grained or fine grained. To make this determination, soil sample is spread on a flat surface. If more than half of the particles are visible to the naked eye, then it is classified as coarse grained or otherwise it is classified as fine grained. If the soil is coarse grained, follow the procedures outlined under the heading coarse grained soil; if the soil is fine grained, follow the procedure mentioned under the heading of fine grained soil.

**Coarse Grained Soil:**

Once the soil has been determined as coarse grained, further examination is required to determine the grain size distribution, the grain shape and gradation of coarse grained soils. Coarse grained soil is classified as cobble or sand depending on whether more than half of the coarse fraction is of cobble size (76 mm or larger) or sand size (5 mm to 0.074 mm). Soil particles can also be described according to a characteristic shape.

Particle shape may vary from angular to round to flat or elongated. Coarse grained soil may be described as well graded, poorly graded or gap graded. A soil is said to well grade if it has a good representation of all grain sizes. If the soil grains are approximately of same size, then the sample is described as poorly graded. A soil is said to be gap graded if the intermediate grain sizes are absent

**Fine Grained Soil:**

Following field tests are performed to classify fine grained soil or for the fine fraction of coarse grained soil

**(i) Dilatancy test:**

Prepare a part of moist soil having a volume equivalent to a 25 mm cube by adding enough water to make the soil soft but not-sticky. Place the pat in the open palm of one hand and shake horizontally by striking against the other hand several times. If the reaction is positive, water appears on the surface of the pat giving a glossy appearance. On squeezing the sample between the fingers the water and glossiness disappear from the surface, the soil becomes stiff and cracks. The phenomenon of appearance of water on the surface of soil on shaking and disappearance on squeezing, followed by cracking is called as “dilatancy”. The rapidity of appearance and disappearance of water from the surface of the soil help to identify the character of fines in the soil. Table 3.6 shows character of fines in soil w.r.t. the positive reactions.

**(ii) Dry strength test:**

Prepare a part of soil to the consistency of putty by adding water. Allow the pat to dry by oven, sun or air. The strength is tested by breaking and crumbling the dry pat between the fingers. Dry strength of soil increases with increasing plasticity. Clays have high dry strength and silts have slight dry strength.

**(iii) Toughness test:**

Take a part of soil to the consistency of putty, add water or allow drying as necessary. Roll the soil between the palms into a thread of 3 mm diameter. Fold the thread of soil and repeat the procedure a number of times till the thread starts crumbling when rolled into 3 mm diameter. The crumbled pieces are lumped together and subjected to kneading until the lump crumbles. The threads are stiffer and lumps are tougher at plastic limit for soils having higher clay contents.

**(iv) Dispersion test:**

Pour small quantity of soil in a jar of water. Shake the jar containing soil and water and allow the soil to settle. The coarser particles settle first followed by finer ones. Sands settle in about 30 to 60 secs, silts settle in 30 to 60 mins and clay particles remains in suspension for at least several hours.



**(v) Bite test:**

Take a pinch of soil and place between the teeth and grind lightly. Fine sand is felt gritty. Silt have rough feeling but do not stick to the teeth, clays have smooth feeling and stick to the teeth.

**(vi) Colour and Odour test:**

Organic soils have darker colours like dark grey, dark brown etc. and a musty odour. The odour can be more noticeable by heating a wet sample. Inorganic soils have clean, bright colours like light grey, brown, red, yellow or white.

**5.0 SOIL WATER AND WATER FLOW****INTRODUCTION**

All soils are permeable materials, water being free to flow through the interconnected pores between the solid particles. The pressure of the pore water is measured relative to atmospheric pressure and the level at which the pressure is atmospheric (i.e. zero) is defined as the water table (WT) or the phreatic surface. Below the water table the soil is assumed to be fully saturated, although it is likely that, due to the presence of small volumes of entrapped air, the degree of saturation will be marginally below 100%.

**SOIL WATER:** Water presence in the voids of soil mass is called soil water. It can be classified in several ways:

Broad classification: 1. Free water 2. Held water

a. Structural water b. Adsorbed water c. Capillary water

Classification on phenomenological basis

1. Ground water 2. Capillary water 3. Adsorbed water 4. Infiltrated water

Classification on structural aspect

1. Pore water 2. Solvate water 3. Adsorbed water 4. Structural water

**Free water:** Water is free to move through a soil mass under the influence of gravity.

**Held water:** It is the part of water held in the soil pores by some force existing within the pores. Such water is not free to move under gravitational force

**Adsorbed water:** Adsorbed water is that water which the soil particles freely adsorb from atmosphere by physical force of attraction and held by force of adhesion. Water in the vicinity of soil particles subjected to an attractive force basically consists of two components. i) Attraction of bipolar water to be electrical charged soil. ii) Attraction of dipolar water to the action in the double layer, cation in turn attract to the particles.

**Structural water:** It is the water chemically combined in the crystal structure of the soil mineral. Structural water cannot be separated or removed and also not removed by oven drying at 105-



110°C. It can be destroyed at higher temperature which will destroy the crystal structure. Infiltrated water Infiltrated water is the portion of surface precipitation which soaks into ground, moving downwards through air containing zones.

**Pore Water:** It is cable of moving under hydrodynamic forces unless restricted in its free movement such as when entrapped between air bubbles or retention by capillary forces. Gravitational and capillary water are the two types of pore water.

**Solvate Water:** The water which forms a hydration shell around soil grains is solvate water. it is subjected to polar electrostatic and binding forces.

**Ground water:** Subsurface water that fills the voids continuously and is subjected to no force other than gravity is known as gravitational water.

**Capillary water:** The minute pores of soil serve as capillary tubes through which the moisture rise above the ground water table. Capillary water is the soil moisture located within the interstices and voids of capillary size of the soil. Capillary water is held in the interstices of soil due to capillary forces. Capillary action or capillarity is the phenomenon of movement of water in the interstices of a soil due to capillary forces. The capillary forces depend upon various factors such as surface tension of water, pressure in water in relation to atmospheric pressure, and the size and conformation of soil pores.

The pores of soil mass may be looked upon as a series of capillary tubes, extending vertically above water table. The rise of water in the capillary tubes, or the fine pores of the soil, is due to the existence of surface tension which pulls the water up against the gravitational force. The height of capillary rise, above the ground water (or free water) surface depends upon the diameter of the capillary tube (or fineness of the pores) and the value of the surface tension. When a capillary tube is inserted in water, the rise of water will take place up to reach the equilibrium. At this stage the rise of water in the tube is stopped.

## 5.1 SOIL SUCTION

The tensile stress in the meniscus circumferences caused in water is called the capillary tension or the capillary potential. The capillary tension or capillary potential is the pressure deficiency, pressure reduction or negative pressure in the pore water (or the pressure below atmospheric) by which water is retained in a soil mass. It decreases linearly from a maximum value of  $h_c \gamma_w$  at the level of the meniscus to zero value at the free water surface. The pressure deficiency in the held water is also termed as soil suction or suction pressure. Soil suction is measured by the height  $h_c$  in centimeters to which a water column could be drawn by suction in a soil mass free from external stress. The common logarithm of this height (cm) or pressure ( $\text{g/cm}^2$ ) is known as the pF value (Schofield, 1935):  $\text{pF} = \log_{10} (h_c)$  Thus, a pF value of 2 represents a soil suction of 100 cm of water or suction pressure and capillarity of  $100 \text{ g/cm}^2$ .

### 5.1.1 *Factors affecting soil suction*

#### 1. *Particle size of soil*

Smaller the size of the particles, smaller will be pore size resulting greater capillary rise and hence greater suction

#### 2. *Water content*

Smaller the water content, greater will be soil suction and attains its maximum value when the soil is dry condition.

#### 3. *Plasticity index of soil mass*

Soil suction will be greater in a soil which has more plasticity index.

#### 4. *Soil structure*

Changes in the soil structure result in the change in the size of the interstices and hence soil suction will change

#### 5. *History of wetting and drying*

For a same soil, suction is greater during drying cycle than during wetting cycle

#### 6. *Soil density*

Increase in density of soil results in decrease in the size of pores of the soil and hence increase in suction. At the loose state the pore sizes will be more resulting a decrease in soil suction.

#### 7. *Temperature*

Rise in temperature results in decrease in surface tension and hence decrease in suction.

#### 8. *Angle of contact*

Soil suction decreases with increase in the angle of contact and will be maximum when angle of contact becomes zero.

#### 9. *Dissolved salts in water*

Impurities such as dissolved salts etc increase the surface tension resulting in increase in soil suction.

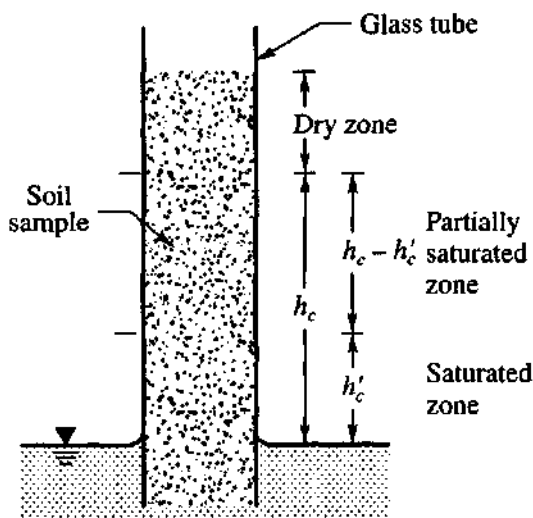
### 5.2 **CAPILLARITY PRESSURE:**

The magnitude of the pressure is the same at all height above the free water surface. The capillarity pressure transferred from grain to grain called as inter angular or effective pressure.

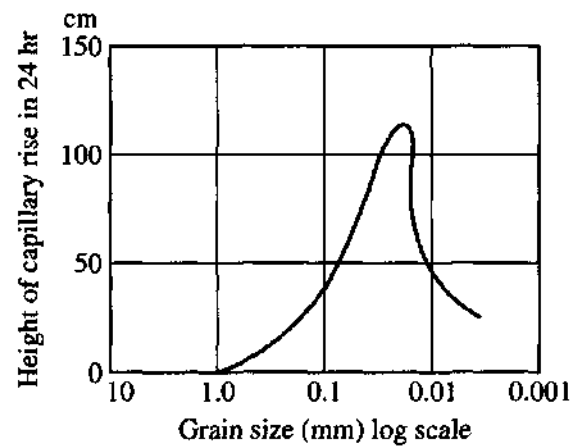
**Capillary action (or) capillarity:** It is the phenomenon of movement of water in the interstices of a soil due to capillary forces. The capillary forces depend upon various factors such as surface tension of water, pressure in water in relation to atmospheric pressure and size and conformation of soil pores.

## Capillary Rise of Water in Soils

In contrast to capillary tubes the continuous voids in soils have a variable width. They communicate with each other in all directions and constitute an intricate network of voids. When water rises into the network from below, the lower part of the network becomes completely saturated. In the upper part, however, the water occupies only the narrowest voids and the wider areas remain filled with air. Fig. shows a glass tube filled with fine sand. Sand would remain fully saturated only up to a height  $h'$  which is considerably smaller than  $h_c$ . A few large voids may effectively stop capillary rise in certain parts. The water would rise, therefore, to a height of  $h_c$  only in the smaller voids. The zone between the depths  $(h_c - h')$  will remain partially saturated.



(a) Height of capillary rise



(b) Rate of capillary rise in soil consisting of uniform quartz powder

Fig.5.1: Capillary rise

The height of the capillary rise is greatest for very fine grained soils materials, but the rate of rise in such materials is slow because of their low permeability. Fig.5.1 (b) shows the relationship between the height of capillary rise in 24 hours and the grain size of a uniform quartz powder. This clearly shows that the rise is a maximum for materials falling in the category of silts and fine sands.

As the effective grain size decreases, the size of the voids also decreases, and the height of capillary rise increases. A rough estimation of the height of capillary rise can be determined from the equation,

$$h_c = \frac{C}{eD_{10}} \text{-----} \quad \text{Eq.5.1}$$

in which  $e$  is the void ratio,  $D_{10}$  is Hazen's effective diameter in centimeters, and  $C$  is an empirical constant which can have a value between 0.1 and 0.5 sq. cm.

### 5.2.1 Capillary Siphoning

The capillary phenomenon which enables raising the water in capillary tube against the force of gravity can also be taken place in voids of dry soil. In case of the earth dam, this may create serious problems. Capillary forces are able to raise water against the force of gravity not only into capillary tubes or the voids in columns of dry soil, but also into narrow open channels or V- shaped grooves. If the highest point of the groove is located below the level to which the surface tension can lift the water, the capillary forces will pull the water into the descending part of the groove and will slowly empty the vessel. This process is known as capillary siphoning. The same process may also occur in the voids of soil. For example, water may flow over the crest of an impermeable core in a dam in spite of the fact that the elevation of the free water surface is below the crest of the core as shown in

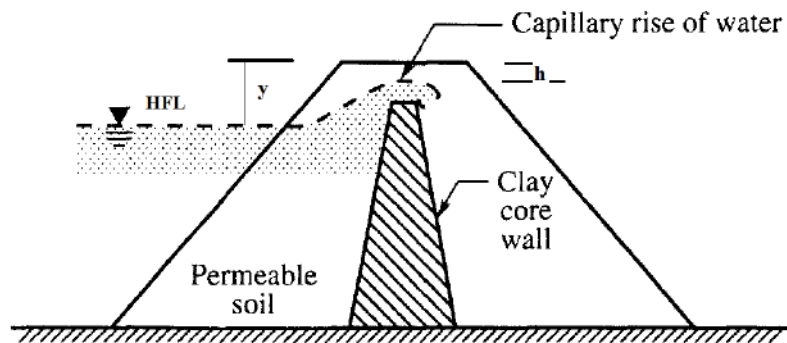


Fig. 5.2: Capillary Siphoning

In an earth dam, the central impervious core provides to check the seepage of water through the body of dam. When the water level in the reservoir reaches HFL, the portion of the upstream of the dam will be saturated. The water level in u/s of the pervious shell will be practically the same as HFL. Now due to capillarity, the water will raise through the height 'h'. If the top of the impervious core is situated at height  $y < h$ , above the HFL, the capillary force will pull the water to the downstream of the dam. This effect will slowly empty the reservoir. This process is known as Capillary Siphoning action in earth dam.

This will not only result in the water loss but also damage the downstream and create instability to the earth dam. So the height of the impervious core is a major factor for design consideration.

**Contact moisture.** Water can also be held by surface tension round the point of contact of two particles (spheres) capillary water in this form is known as contact moisture (or) contact capillary water.

## 6.0 EFFECTIVE STRESS CONCEPTS IN SOIL

At any plane in a soil mass, the total stress or unit pressure  $\sigma$  is the total load per unit area. This pressure may be due to i) self weight of soil ii) over burden on the soil. The total pressure consists of two distinct components: inter granular pressure or effective pressure and the neutral pressure or pore pressure. Effective pressure  $\sigma'$  is the pressure transmitted from particle through their point of contact through the soil mass above the plane. Such a pressure, also termed as inter

granular pressure, is effective in decreasing the voids ratio of the soil mass and in mobilizing its shear strength. The neutral pressure or the pore water pressure or pore pressure is the pressure transmitted through the pore fluid. Therefore, this pressure is also called neutral pressure ( $u$ ). Since the total vertical pressure at any plane is equal to the sum of the effective pressure and pore water pressure we have

$$\sigma = \sigma' + u \text{-----}$$

Eq.6.1

## 6.1 TOTAL AND EFFECTIVE STRESS IN SOILS

When a load is applied to soil, it is carried by the solid grains and the water in the pores.

The **total vertical stress** acting at a point below the ground surface is due to the weight of everything that lies above, including soil, water, and surface loading. Total stress thus increases with depth and with unit weight.

Vertical total stress at depth  $z$ ,  $\sigma_v = \gamma Z$

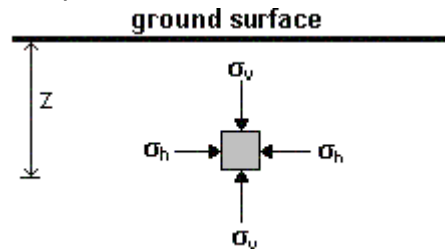


Fig.6.1: Stresses acting on a soil element

At any plane in a soil mass, the total stress or unit pressure  $\sigma$  is the total load per unit area. This pressure may be due to i) self weight of soil ii) over burden on the soil. The total pressure consists of two distinct components: inter granular pressure or effective pressure and the neutral pressure or pore pressure. Effective pressure  $\sigma'$  is the pressure transmitted from particle through their point of contact through the soil mass above the plane. Such a pressure, also termed as inter granular pressure, is effective in decreasing the voids ratio of the soil mass and in mobilizing its shear strength. The neutral pressure or the pore water pressure or pore pressure is the pressure transmitted through the pore fluid.

Therefore, this pressure is also called neutral pressure ( $u$ ). Since the total vertical pressure at any plane is equal to the sum of the effective pressure and pore water pressure we have,

The pressure of water in the pores of the soil is called **pore water pressure ( $u$ )**. The magnitude of pore water pressure depends on:

- the depth below the water table.
- the conditions of seepage flow.

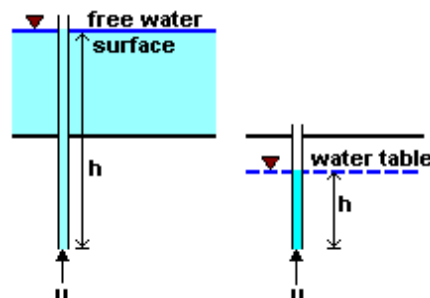


Fig.6.2: Pore water pressure

Under hydrostatic conditions, no water flow takes place, and the pore pressure at a given point is given by

$$u = \gamma_w h \text{-----} \quad \text{Eq.6.2}$$

Where,  $h$  = depth below water table or overlying water surface

It is convenient to think of pore water pressure as the pressure exerted by a column of water in an imaginary standpipe inserted at the given point.

The natural level of ground water is called the **water table** or the **phreatic surface**. Under conditions of no seepage flow, the water table is horizontal. The magnitude of the pore water pressure at the water table is zero. Below the water table, pore water pressures are positive.

The **principle of effective stress** was enunciated by **Karl Terzaghi** in the year 1936. This principle is valid only for saturated soils, and consists of two parts:

1. At any point in a soil mass, the effective stress (represented by  $\bar{\sigma}$  or  $\sigma'$ ) is related to total stress ( $\sigma$ ) and pore water pressure ( $u$ ) as

$$\sigma' = \sigma - u \text{-----} \quad \text{Eq.6.3}$$

Both the total stress and pore water pressure can be measured at any point.

2. All measurable effects of a change of stress, such as compression and a change of shearing resistance, are exclusively due to changes in effective stress.

Compression =  $f_1(\sigma)$

Shear Strength =  $f_2(\sigma)$

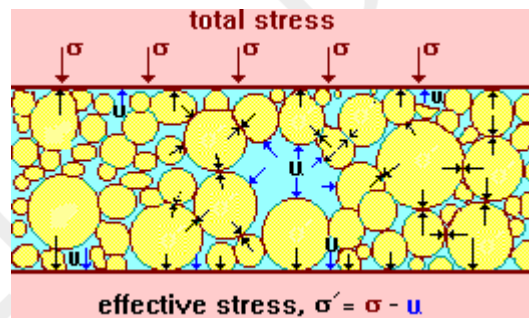


Fig. 6.3: Total Stress and Effective Stress acting on Soil

In a saturated soil system, as the voids are completely filled with water, the pore water pressure acts equally in all directions.

The effective stress is not the exact contact stress between particles but the distribution of load carried by the soil particles over the area considered. It cannot be measured and can only be computed.

If the total stress is increased due to additional load applied to the soil, the pore water pressure initially increases to counteract the additional stress. This increase in pressure within the pores might cause water to drain out of the soil mass, and the load is transferred to the solid grains. This will lead to the increase of effective stress.

Above the water table, when the soil is saturated, pore pressure will be negative (less than atmospheric). The height above the water table to which the soil is saturated is called

the **capillary rise**, and this depends on the grain size and the size of pores. In coarse soils, the capillary rise is very small.

## 6.2 STRESSES WHEN NO FLOW TAKES PLACE THROUGH THE SATURATED SOIL MASS

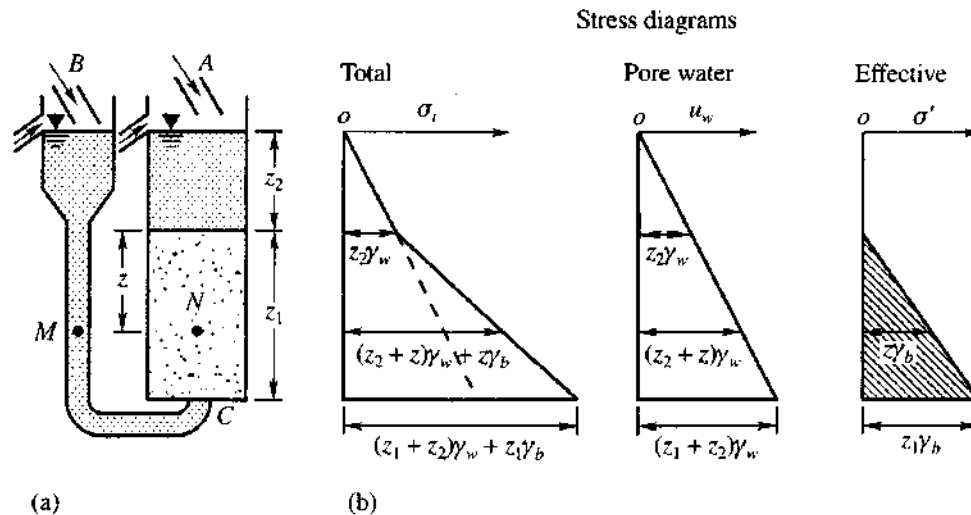


Fig.6.4: Stresses when no flow takes place

The above figures show that the container A is filled with sand to a depth  $z_1$  and water to a depth  $z_2$  above the sand surface. A flexible tube connects the bottom of the container A to another container B. The water levels are kept constant in these two containers.

The water surfaces in both the containers in Fig. 6.4(a) are kept at the same level. Under this condition, no flow takes place from one container to another. Consider two points M and N as shown in the figure on a horizontal plane. The water pressure at M should be equal to the pressure at N according to the laws of hydraulics. Therefore, the water pressure at N,  $u_z = (z + z_2)\gamma_w$

The pressure  $u_z$  is termed as the pore water pressure acting on the grains at depth  $z$  from the surface of the sample. However, the total pressure at point N is due to the water head plus the weight of the submerged soil above N. If  $\gamma_b$  is the submerged unit weight of the soil, the total pressure at N is

$$\sigma_z = z\gamma_b + (z + z_2)\gamma_w \text{-----} \quad \text{Eq.6.4}$$

The inter-granular or effective pressure at the point N is the difference between the total and the pore water pressures. Therefore, the effective pressure

$$\sigma'_z = \sigma_z - u_z = z\gamma_b \text{-----} \quad \text{Eq.6.5}$$

The above equation clearly demonstrates that the effective pressure is independent of the depth of water  $z_2$  above the submerged soil surface.

## 6.3 STRESSES WHEN FLOW TAKES PLACE THROUGH THE SOIL FROM TOP TO BOTTOM

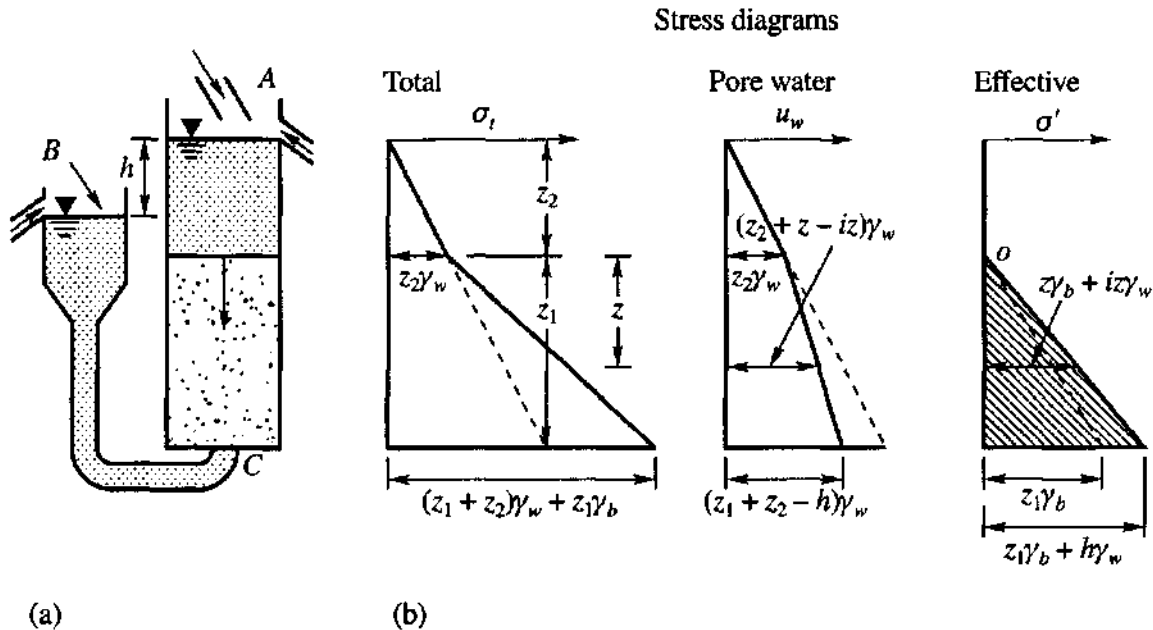


Fig. 6.5: Stresses when flow takes place in downward direction

In Fig. 6.5 (a) the water surface in container B is kept at  $h$  units below the surface in A. This difference in head permits water to flow from container A to B. Since container B with the flexible tube can be considered as a piezometer tube freely communicating with the bottom of container A, the piezometric head or the pore water pressure head at the bottom of container A is  $(z_1 + z_2 - h)$ . Therefore, the pore water pressure  $u_c$  at the bottom level is

$$u_c = (z_1 + z_2 - h)\gamma_w \quad \text{Eq.6.6}$$

As per Fig.6.5(a), the pore water pressure at the bottom of container A when no flow takes place through the soil sample is

$$u_c = (z_1 + z_2)\gamma_w \quad \text{Eq.6.7}$$

It is clear from the above expressions, that there is a decrease in pore water pressure to the extent of  $\gamma_w$  when water flows through the soil sample from top to bottom. It may be understood that this decrease in pore water pressure is not due to velocity of the flowing water. The value of the velocity head  $\frac{v^2}{2g}$  is a negligible quantity even if we take the highest velocity of flow that is encountered in natural soil deposits. As shown in Fig. 6.5. (a), the total pressure  $\sigma_c$  at the bottom of the container in this case also remains the same. Therefore,

$$\sigma_c = z_1\gamma_b + (z_1 + z_2)\gamma_w \quad \text{Eq.6.8}$$

The effective pressure at the bottom of the container is

$$\sigma'_c = \sigma_c - u_c = z_1\gamma_b + h\gamma_w \quad \text{Eq.6.9}$$

Equation indicates that in this case there is an increase in the effective pressure by  $h\gamma_w$  at the bottom of the container A as compared to the earlier case. The effective pressure at the top surface of the sample is zero as before. Therefore, the effective pressure at any depth  $z$  can be written as



$$\sigma'_z = \sigma'_c(z) = (z \gamma_b + h \gamma_w) \left( \frac{z}{z_1} \right) \text{-----} \quad \text{Eq.6.10}$$

$$= z \gamma_b + \frac{h z \gamma_w}{z_1} \text{-----} \quad \text{Eq.6.11}$$

The above equation indicates that  $\frac{h z \gamma_w}{z_1}$  is the increase in the effective pressure as the water flows from the surface to a depth  $z$ . This increase in effective pressure due to the flow of water through the pores of the soil is known as *seepage pressure*. It may be noted that  $h$  is the total loss of head as the water flows from the top surface of the sample to a depth  $z_1$ .

The corresponding loss of head at depth  $z$  is  $(z/z_1)h$ . Since  $(z/z_1) = i$  the *hydraulic gradient*, the loss of head at depth  $z$  can be expressed as  $iz$ . Therefore the seepage pressure at any depth may be expressed as  $iz \gamma_w$ . The effective pressure at depth  $z$  can be written as

$$\sigma'_z = z \gamma_b + i z \gamma_w \text{-----} \quad \text{Eq.6.12}$$

The distribution of pore water and effective pressures are shown in Fig. 6.5.(b). In normal soil deposits when flow takes place in the direction of gravity there will be an increase in the effective pressure.

#### 6.4 STRESSES WHEN FLOW TAKES PLACE THROUGH THE SOIL FROM BOTTOM TO TOP

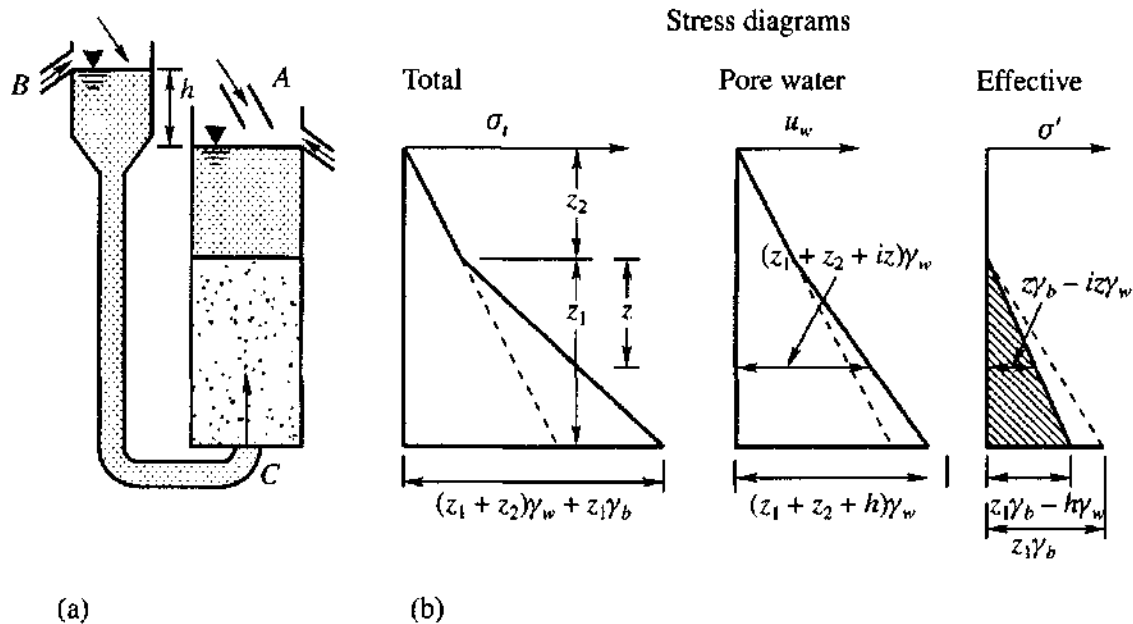


Fig. 6.6: Stresses when flow takes place in upward direction

In Fig. 6.6 (a), the water surface in container B is kept above that of A by  $h$  units. This arrangement permits water to flow upwards through the sample in container A. The total piezometric or the pore water head at the bottom of the sample is given by  $(z_1 + z_2 + h)$ .

Therefore, the pore water pressure  $u_c$  at the bottom of the sample is

$$u_c = (z_1 + z_2 + h)\gamma_w \text{-----} \quad \text{Eq.6.13}$$

As before the total pressure head  $\sigma_c$  at the bottom of the sample is

$$\sigma_c = z_1\gamma_b + (z_1 + z_2 + h)\gamma_w \text{-----} \quad \text{Eq.6.14}$$

The effective pressure at the bottom of sample is, therefore,

$$\begin{aligned} \sigma'_c &= \sigma_c - u_c = z_1\gamma_b + (z_1 + z_2 + h)\gamma_w - (z_1 + z_2 + h)\gamma_w \\ &= z_1\gamma_b - z\gamma_w \text{-----} \quad \text{Eq.6.15} \end{aligned}$$

This equation indicates that there is a decrease in the effective pressure due to upward flow of water. At any depth  $z$ ,  $z\gamma_b$  is the pressure of the submerged soil acting downward and  $iz\gamma_b$  is the seepage pressure acting upward. The effective pressure reduces to zero when these two pressures balance. This happens when

$$\sigma'_c = \sigma_c - u_c = 0 \text{ Or } z_1\gamma_b - z\gamma_w = 0 \text{ and hence } i = i_c = \frac{z_1\gamma_b}{z\gamma_w} \text{-----} \quad \text{Eq.6.16}$$

This indicates that the effective pressure reduces to zero when the hydraulic gradient attains a maximum value which is equal to the ratio of the submerged unit weight of soil and the unit weight of water. This gradient is known as the *critical hydraulic gradient*  $i_c$ . In such cases, cohesionless soils lose all of their shear strength and bearing capacity and a visible agitation of soil grains is observed. This phenomenon is known as *boiling* or a **quick sand condition**.

Hence critical hydraulic gradient can be expressed as

$$i_c = \frac{G-1}{1+e} \text{-----} \quad \text{Eq.6.17}$$

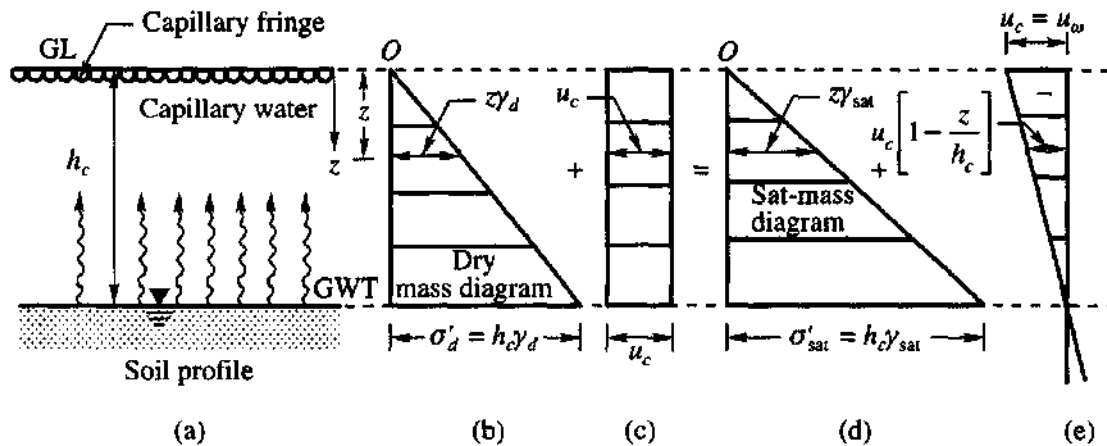
Quick conditions are common in excavations below the ground water table. This can be prevented by lowering the ground water elevation by pumping before excavation. Quick conditions occur most often in fine sands or silts and cannot occur in coarse soils. The larger the particle size, the greater is the porosity. To maintain a critical gradient of unity, the velocity at which water must be supplied at the point of inflow varies as the permeability. Therefore a quick condition cannot occur in a coarse soil unless a large quantity of water can be supplied.

## 6.5 EFFECTIVE PRESSURE DUE TO CAPILLARY WATER RISE IN SOIL

Since the capillary pressure inside a soil mass is below atmospheric pressure, it draws the grains of soils closer to each other at all points where the menisci touch the soil grains. Inter-granular pressure of this type is called *capillary pressure*. The effective or inter-granular pressure at any point in a soil mass can be expressed by

$$\sigma'_z = \sigma - u_w \text{-----} \quad \text{Eq.6.18}$$

where  $\sigma$  is the total pressure,  $\sigma'_z$  is the effective or the inter-granular pressure and  $u_w$  is the pore water pressure. When the water is in compression  $u_w$  is positive, and when it is in tension  $u_w$  is negative. Since  $u_w$  is negative in the capillary zone, the inter-granular pressure is increased by  $u_w$ . The equation, therefore, can be written as

Fig.6.7: Effect of capillary pressure  $u_c$  on soil vertical stress diagram

$$\sigma' = \sigma + u_w$$

Eq.6.19

The increase in the inter-granular pressure due to capillary pressure acting on the grains leads to greater strength of the soil mass.

## 6.6 Important of Effective Stress

At any point within the soil mass, the magnitudes of both total stress and pore water pressure are dependent on the ground water position. With a shift in the water table due to seasonal fluctuations, there is a resulting change in the distribution in pore water pressure with depth.

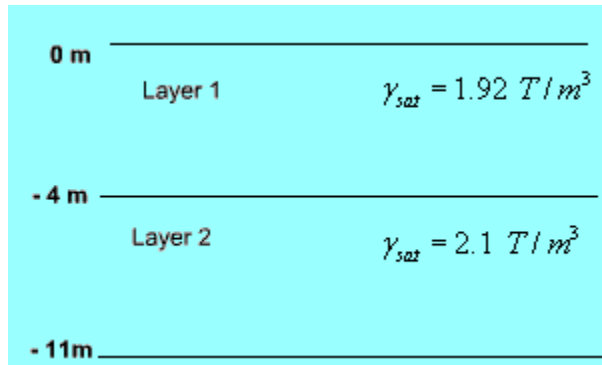
Changes in water level **below ground** result in changes in effective stresses below the water table. A rise increases the pore water pressure at all elevations thus causing a decrease in effective stress. In contrast, a fall in the water table produces an increase in the effective stress.

Changes in water level **above ground** do not cause changes in effective stresses in the ground below. A rise above ground surface increases both the total stress and the pore water pressure by the same amount, and consequently effective stress is not altered.

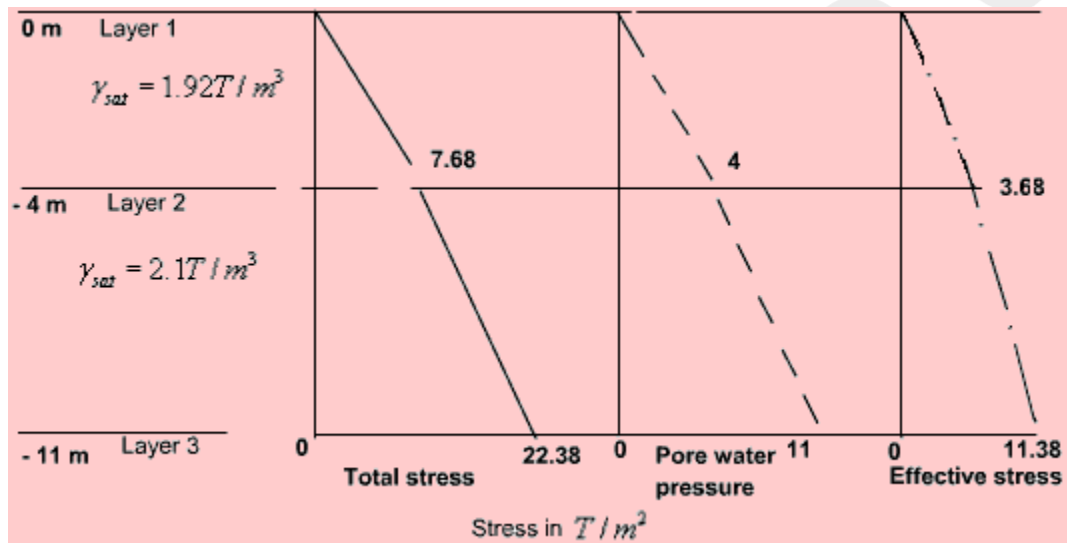
If both total stress and pore water pressure change by the same amount, the effective stress remains constant.

Total and effective stresses must be distinguishable in all calculations. Ground movements and instabilities can be caused by changes in total stress, such as caused by loading by foundations and unloading due to excavations. They can also be caused by changes in pore water pressures, such as failure of slopes after rainfall.

**Example 1:** For the soil deposit shown below, draw the total stress, pore water pressure and effective stress diagrams. The water table is at ground level.



**Solution:**



**Total stress**

At - 4m,  $\sigma = 1.92 \times 4 = 7.68 \text{ T/m}^2$

At -11m,  $\sigma = 7.68 + 2.1 \times 7 = 22.38 \text{ T/m}^2$

**Pore water pressure**

At - 4 m,  $u = 1 \times 4 = 4 \text{ T/m}^2$

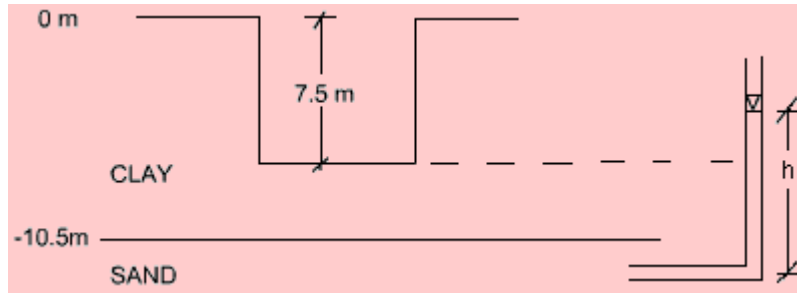
At -11 m,  $u = 1 \times 11 = 11 \text{ T/m}^2$

**Effective stress**

At - 4 m,  $\bar{\sigma} = 7.68 - 4 = 3.68 \text{ T/m}^2$

At -11m,  $\bar{\sigma} = 22.38 - 11 = 11.38 \text{ T/m}^2$

**Example 2:** An excavation was made in a clay stratum having  $\gamma_t = 2 \text{ T/m}^3$ . When the depth was 7.5 m, the bottom of the excavation cracked and the pit was filled by a mixture of sand and water. The thickness of the clay layer was 10.5 m, and below it was a layer of pervious water-bearing sand. How much was the artesian pressure in the sand layer?



**Solution:**

When the depth of excavation was 7.5 m, at the interface of the CLAY and SAND layers, the effective stress was equal to zero.

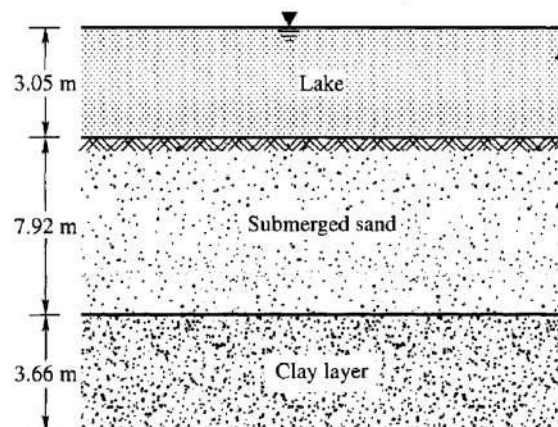
Downward pressure due to weight of clay = Upward pressure due to artesian pressure

$$(10.5 - 7.5) \gamma_t = \gamma_w h, \text{ where } h = \text{artesian pressure head}$$

$$3 \times 2 = 1 \times h$$

$$\therefore h = 6 \text{ m} = 0.6 \text{ kg/cm}^2 \text{ or } 6 \text{ T/m}^2 \text{ artesian pressure}$$

**Example 3** A clay layer 3.66 m thick rests beneath a deposit of submerged sand 7.92 m thick. The top of the sand is located 3.05 m below the surface of a lake. The saturated unit weight of the sand is  $19.62 \text{ kN/m}^3$  and of the clay is  $18.36 \text{ kN/m}^3$  Compute (a) the total vertical pressure, (b) the pore water pressure, and (c) the effective vertical pressure at mid height of the clay layer (Refer to Fig. Ex.3.)



Sol:

(a) *Total pressure*

The total pressure cr, over the midpoint of the clay is due to the saturated weights of clay and

sand layers plus the weight of water over the bed of sand, that is

$$\sigma = 3.66/2 \times 18.36 + 7.92 \times 19.62 + 3.05 \times 9.81 = 33.6 + 155.4 + 29.9 = 218.9 \text{ kN/m}^2$$

(b) Pore water pressure is due to the total water column above the midpoint.

That is

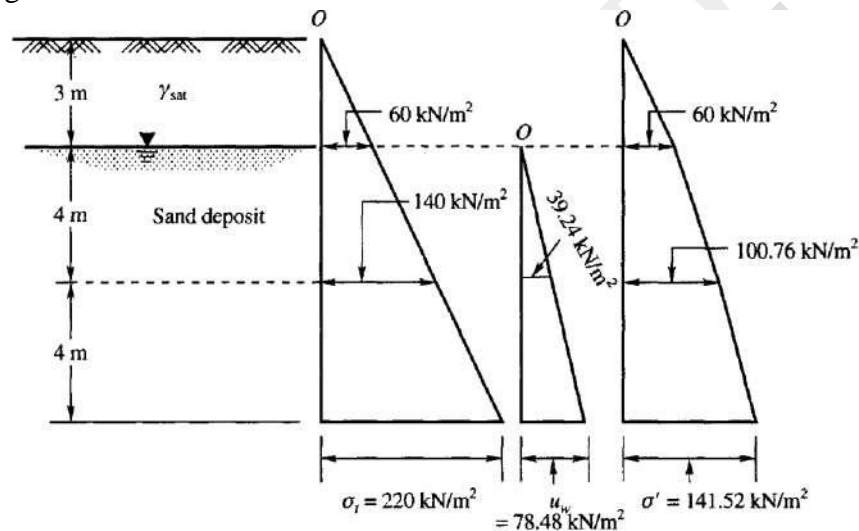
$$u = 3.66/2 \times 9.81 + 7.92 \times 9.81 + 3.05 \times 9.81 = 125.6 \text{ kN/m}^2$$

(c) Effective vertical pressure

$$\sigma - u = 218.9 - 125.6 = 93.3 \text{ kN/m}^2$$

#### Example 4

The water table is located at a depth of 3.0 m below the ground surface in a deposit of sand 11.0 m thick (Fig. Ex. 4). The sand is saturated above the water table. The total unit weight of the sand is  $20 \text{ kN/m}^3$ . Calculate the (a) the total pressure, (b) the pore water pressure and (c) the effective pressure at depths 0, 3.0, 7.0, and 11.0 m from the ground surface, and draw the pressure distribution diagram.



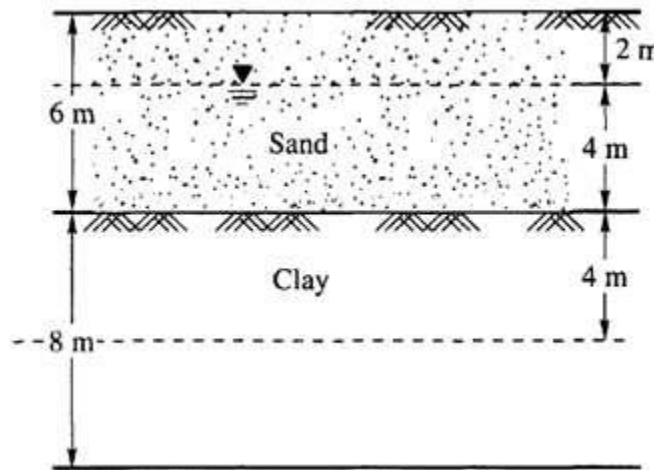
The calculation of total and effective stresses at different depth are shown in the table

Depth (m)	Total pressure $\sigma_t$ (kN/m <sup>2</sup> )	Pore water pressure $u_w$ (kN/m <sup>2</sup> )	Effective pressure $\sigma'$ (kN/m <sup>2</sup> )
0	0	0	0
3	$3 \times 20 = 60$	0	60
7	$7 \times 20 = 140.00$	$4 \times 9.81 = 39.24$	100.76
11	$11 \times 20 = 220.00$	$8 \times 9.81 = 78.48$	141.52

#### Problem No.1

A clay stratum 8.0 m thick is located at a depth of 6 m from the ground surface. The natural moisture content of the clay is 56% and  $G = 2.75$ . The soil stratum between the ground surface and the clay consists of fine sand. The water table is located at a depth of 2 m below the ground

surface. The submerged unit weight of fine sand is  $10.5 \text{ kN/m}^3$ , and its moist unit weight above the water table is  $18.68 \text{ kN/m}^3$ . Calculate the effective stress at the center of the clay layer.

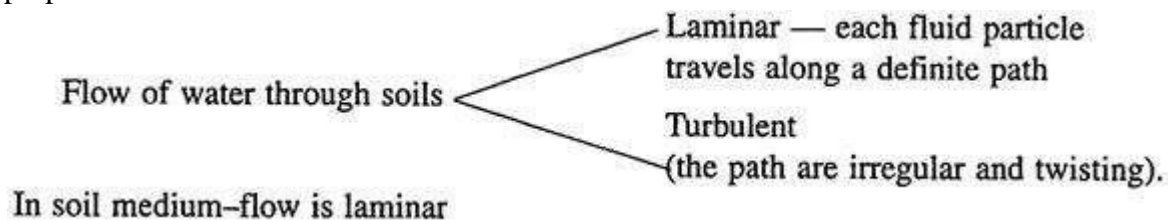


## 7.0 PERMEABILITY OF SOIL

### *Definition of Permeability:*

It is defined as the property of a porous material which permits the passage or seepage of water (or other fluids) through its interconnecting voids.

A material having continuous voids is called permeable. Gravels are highly permeable while stiff clay is the least permeable, and hence such a clay may be termed impermeable for all practical purpose.



**The study of seepage of water through soil is important for the following engineering problems:**

1. Determination of rate of settlement of a saturated compressible soil layer.
2. Calculation of seepage through the body of earth dams and stability of slopes for highways.
3. Calculation of uplift pressure under hydraulic structure and their safety against piping.
4. Groundwater flow towards well and drainage of soil.

### **7.1 Darcy's Law (1856) of Permeability:**

For laminar flow conditions in a saturated soil, the rate of the discharge per unit time is proportional to the hydraulic gradient.

$$q = kiA$$

Eq.7.1



$$v = q/A = ki \dots$$

Eq.7.2

Where  $q$  = discharge per unit time

$A$  = total cross-sectional area of soil mass, perpendicular to the direction of flow

$i$  = hydraulic gradient

$k$  = Darcy's coefficient of permeability

$v$  = velocity of flow or average discharge velocity

### 7.1.1 Coefficient of permeability (or) permeability

It is defined as the average velocity of flow that will occur through the total cross-sectional area of soil under unit hydraulic gradient. The coefficient of permeability is denoted as  $K$ . It is usually expressed as cm/sec (or) m/day (or) feet/day.

When hydraulic gradient is unity,  $k$  is equal to  $V$ . Thus, the coefficient of permeability, or simply permeability is defined as the average velocity of flow that will occur through the total cross-sectional area of soil under unit hydraulic gradient. Dimensions are same as of velocity, cm/sec.

### 7.1.2 Discharge and seepage velocities

Figure 7.1 shows a soil sample of length  $L$  and cross-sectional area  $A$ . The sample is placed in a cylindrical horizontal tube between screens. The tube is connected to two reservoirs  $R_1$  and  $R_2$  in which the water levels are maintained constant. The difference in head between  $R_1$  and  $R_2$  is  $h$ . This difference in head is responsible for the flow of water. Since Darcy's law assumes no change in the volume of voids and the soil is saturated, the quantity of flow past sections  $AA$ ,  $BB$  and  $CC$  should remain the same for steady flow conditions. We may express the equation of continuity as follows

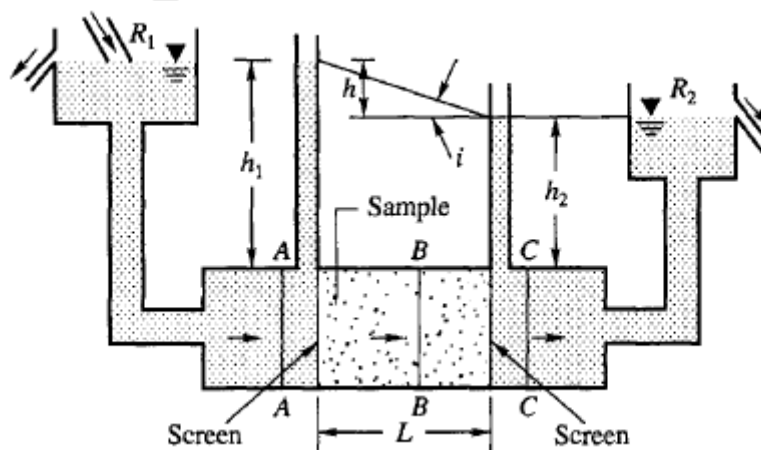


Fig.7.1: Flow of water through a sample soil

$$q_{aa} = q_{bb} = q_{cc} \dots \dots \dots \text{Eq.7.3}$$

If soil mass is divided into two parts soil solid and void space, then the area available for passage of water is only through the area of voids  $A_v$ . If  $v_s$  is the velocity of flow in voids and  $v$  is the average velocity across the section then we have

$$A_v v_s = A v \text{-----} \quad \text{Eq.7.4}$$

$$\text{Or } v_s = \frac{A}{A_v} v \text{-----} \quad \text{Eq.7.5}$$

$$\text{But since, } \frac{A}{A_v} = \frac{1}{n} = \frac{1+e}{e} \text{-----} \quad \text{Eq.7.6}$$

$$\text{So } v_s = \frac{v}{n} = \frac{1+e}{e} v \text{-----} \quad \text{Eq.7.7}$$

Since  $\frac{1+e}{e}$  is always greater than unity,  $v_s$  is always greater than  $v$ .

### 7.1.3 Validity of Darcy's Law:

In accordance with the Darcy's Law, the velocity of flow through soil mass is directly proportion to the hydraulic gradient for laminar flow condition only. It is expected that the flow to be always laminar in case of fine-grained soil deposits because of low permeability and hence low velocity of flow.

So  $v = ki$

Where  $i$  is the hydraulic gradient and  $k$  is a constant known as coefficient of permeability.

However, in case of sands and gravels flow will be laminar up to a certain value of velocity for each deposit and investigations have been carried out to find a limit for application of Darcy's law.

According to researchers, flow through sands will be laminar and Darcy's law is valid so long as Reynolds number expressed in the form is less than or equal to unity as shown below

$$\frac{v D_a \gamma_w}{\eta g} \leq 1 \text{-----} \quad \text{Eq.7.8}$$

Where  $v$  = velocity of flow in cm/sec

$D_a$  = size of particles (average) in cm.

It is found that the limiting value of Reynolds number taken as 1 is very approximate as its actual value can have wide variation depending partly on the characteristic size of particles used in the equation.

## 7.2 FACTORS AFFECTING PERMEABILITY ARE

1. Size of soil particle
2. Specific Surface Area of Soil Particle
3. Shape of soil particle
4. Void ratio
5. Soil structure
6. Degree of saturation
7. Water properties
8. Temperature
9. Adsorbed water

## 10. Organic Matter

### 1. Size of Soil Particle

Permeability varies according to size of soil particle. If the soil is coarse grained, permeability is more and if it is fine grained, permeability is low. The relation between coefficient of permeability ( $k$ ) and particle size ( $D$ ) can be shown from equation (1) as follows.

$$k \propto D^2$$

### 2. Specific Surface Area of Particles

Specific surface area of soil particles also affects the permeability. Higher the specific surface area lower will be the permeability.

$$k \propto \frac{1}{\text{Specific Surface Area}}$$

### 3. Shape of Soil Particle

Rounded Particles will have more permeability than angular shaped. It is due to specific surface area of angular particles is more compared to rounded particles.

### 4. Void Ratio

In general, Permeability increases with void ratio. But it is not applicable to all types of soils. For example, Clay has high void ratio than any other types of soil but permeability for clays is very low. This is due to, the flow path through voids in case of clays is extremely small such that water cannot permit through this path easily.

The relation between coefficient of permeability and void ratio can be expressed as

$$\text{For Clay } k \propto \frac{e^3}{1+e}$$

Where,  $C$  = Shape of the flow path,

$e$  = Void ratio.

For coarse grained soil, “ $C$ ” can be neglected. Hence  $k \propto \frac{e^3}{1+e}$

### 5. Soil Structure

Structure of any two similar soil masses at same void ratio need not be same. It varies according to the level of compaction applied. If a soil contains flocculated structure, the particles are in random orientation and permeability is more in this case.

If the soil contains dispersed structure, the particles are in face to face orientation hence, permeability is very low. The permeability of stratified soil deposits also varies according to the flow direction. If the flow is parallel, permeability is more. If it is perpendicular, permeability is less.

### 6. Degree of Saturation

Partially saturated soil contains air voids which are formed due to entrapped air or gas released from the percolating fluid or water. This air will block the flow path thereby reduces the permeability. Fully saturated soil is more permeable than partially saturated soil.

### 7. Water Properties

Various properties of water or fluid such as unit weight and viscosity also effects the permeability. However, unit weight of water will not affect much since it does not change much with temperature.

But when temperature is increased viscosity decreases rapidly. From equation (1), permeability increase when viscosity decreases.

$$k \propto \frac{\gamma_w}{\mu}$$

## 8. Temperature

Temperature also affects the permeability in soils. The permeability is inversely proportional to the viscosity of the fluid. It is known that viscosity varies inversely to the temperature. Hence, Permeability is directly related to temperature.

Greater the temperature, higher will be the permeability. That is the reason; seepage is more in summer seasons than in winter.

$$k \propto \frac{1}{\mu} \propto \text{temperature}$$

## 9. Adsorbed Water

Adsorbed water is the water layer formed around the soil particle especially in the case of fine-grained soils. This reduces the size of the void space by about 10%. Hence, permeability reduces.

## 10. Organic matter

Presence of organic matter decreases the permeability. This is due to blockage of voids by the organic matter.

### 7.3 PERMEABILITY OF STRATIFIED SOIL DEPOSITS:

Natural soil deposits may exhibit stratification. Each layer may have its own coefficient of permeability, assuming it to be homogeneous. The ‘average permeability’ of the entire deposit will depend upon the direction of flow in relation to the orientation the bedding planes.

Two cases will be considered—the first one with flow perpendicular to the bedding planes and the next with flow parallel to the bedding planes.

#### 7.3.1 Flow Perpendicular to the Bedding Planes

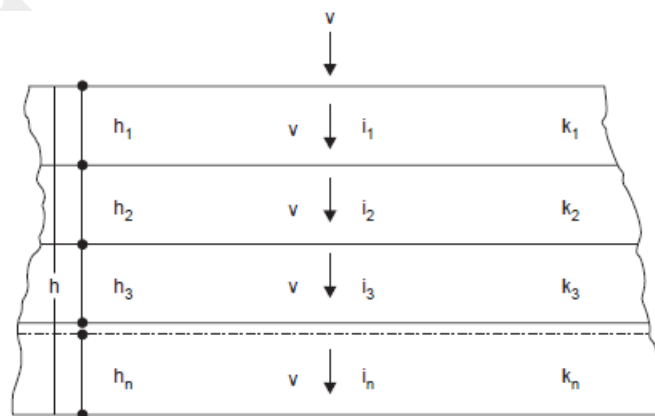


Fig. 7.2: Flow perpendicular to bedding planes

Let  $h_1, h_2, h_3 \dots h_n$  be the thicknesses of each of the  $n$  layers which constitute the deposit, of total thickness  $h$ . Let  $k_1, k_2, k_3 \dots k_n$  be the Darcy coefficients of permeability of these layers respectively. In this case, the velocity of flow  $v$ , and hence the discharge  $q$ , is the same through all the layers, for the continuity of flow.

Let the total head lost be  $\Delta h$  and the head lost in each of the layers be  $\Delta h_1, \Delta h_2, \Delta h_3, \dots$

$$\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3 + \dots + \Delta h_n \quad \text{Eq.7.9}$$

The hydraulic gradients are

$$i_1 = \frac{\Delta h_1}{h_1} \text{ where } i \text{ is the hydraulic gradient for deposit}$$

$$i_2 = \frac{\Delta h_2}{h_2}$$

$$i_n = \frac{\Delta h_n}{h_n} \quad \text{Eq.7.10}$$

Since  $q$  is the same in all the layers, and area of cross-section of flow is the same, the velocity is the same in all layers.

Let  $k_z$  be the average permeability perpendicular to the bedding planes

$$\text{Now } k_z i = k_1 i_1 = k_2 i_2 = \dots = k_n i_n = v \quad \text{Eq.7.11}$$

$$\therefore k_z \frac{\Delta h}{h} = k_1 \frac{\Delta h_1}{h_1} = k_2 \frac{\Delta h_2}{h_2} = \dots = k_n \frac{\Delta h_n}{h_n} = v \quad \text{Eq.7.12}$$

Now substituting the expression for  $\Delta h_1, \Delta h_2, \dots$  in terms of  $v$  in the equation for  $\Delta h$ , we get

$$v h / k_z = \frac{v h_1}{k_1} + \frac{v h_2}{k_2} + \dots + \frac{v h_n}{k_n} \quad \text{Eq.7.13}$$

$$\text{Hence } k_z = \frac{h}{\frac{h_1}{k_1} + \frac{h_2}{k_2} + \dots + \frac{h_n}{k_n}} \quad \text{Eq.7.14}$$

This is the equation for average permeability for flow perpendicular to the bedding planes.

### 7.3.2 Flow Parallel to the Bedding Planes

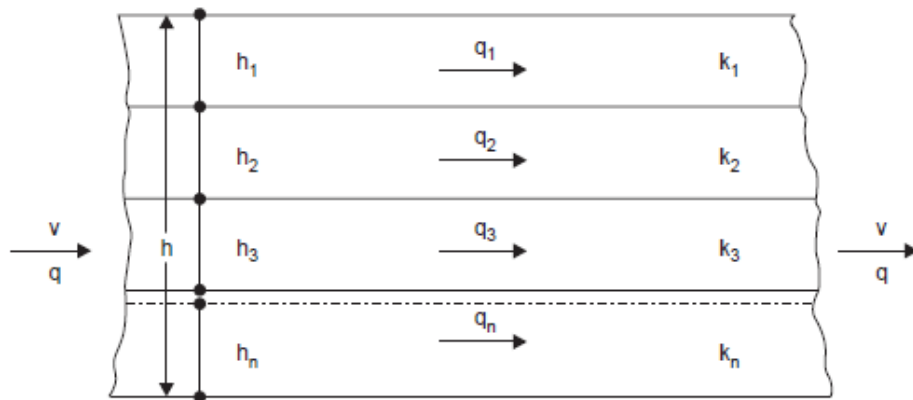


Fig. 7.3: Flow parallel to the bedding planes

Let the flow be parallel to the bedding planes as shown in Figure 7.3

With the same notation as in the first case, the hydraulic gradient  $i$  will be the same for all the layers as for the entire deposit. Since  $v = ki$ , and  $k$  is different for different layers,  $v$  will be different for the layers, say,  $v_1, v_2, \dots, v_n$

Also  $v_1 = k_1 i$ ,  $v_2 = k_2 i$  and so on

Considering unit dimension perpendicular to the plane of the paper, the areas of flow for each layer will be plane of the paper, the areas of flow for each layer will be  $h_1, h_2, \dots, h_n$  respectively, and it is  $h$  for the entire deposit.

The discharge through the entire deposit is equal to the sum of the discharge through the individual layers. Assuming  $k_x$  to be the average permeability of the entire deposit parallel to the bedding planes, and applying the equation:

$$q = q_1 + q_2 + \dots + q_n \text{-----} \quad \text{Eq.7.15}$$

$$\text{We have, } k_x i h = k_1 i h_1 + k_2 i h_2 + \dots + k_n i h_n \text{-----} \quad \text{Eq.7.16}$$

$$\therefore k_x = \left[ \frac{k_1 h_1 + k_2 h_2 + \dots + k_n h_n}{h} \right] \text{-----} \quad \text{Eq.7.17}$$

$$\text{Where } h = h_1 + h_2 + \dots + h_n \text{-----} \quad \text{Eq.7.18}$$

In other words,  $k_x$  is the weighted mean value, the weights being the thickness for each layer.

It can be shown that  $k_x$  is always greater than  $k_z$  for a given situation.

## 7.4 CONSTANT HEAD PERMEABILITY TEST

The constant head permeability test is a laboratory experiment conducted to determine the permeability of soil. The soils that are suitable for this test are sand and gravels. Soils with silt content cannot be tested with this method. The test can be employed to test granular soils either reconstituted or disturbed.

What is Coefficient of Permeability?

The coefficient of permeability,  $k$  is defined as the rate of flow of water under laminar flow conditions through a porous medium area of unit cross section under unit hydraulic gradient.

The coefficient of permeability ( $k$ ) is obtained from the relation

$$k = \frac{qL}{Ah} = \frac{QL}{Aht} \text{-----} \quad \text{Eq.7.19}$$

Where  $q$  = Discharge,  $Q$  = Total volume of water,  $t$  = time period,  $h$  = Head causing flow,  $L$  = Length of specimen,  $A$  = Cross-sectional area.

*Apparatus for Constant Head Permeability Test*

- 1 Permeameter mould, internal diameter = 100mm, effective height = 127.3 mm, capacity = 1000ml.
- 2 Detachable collar, 100mm diameter, 60mm height
- 3 Dummy plate, 108 mm diameter, 12mm thick,
- 4 Drainage base, having porous disc

- 5 Drainage cap having porous disc with a spring attached to the top.
- 6 Compaction equipment such as Proctor's rammer or a static compaction equipment, as specified in IS: 2720 (Part VII)-1965.
- 7 Constant head water supply reservoir
- 8 Vacuum pump
- 9 Constant head collecting chamber
- 10 Stop watch
- 11 Large funnel
- 12 Thermometer
- 13 Weighing balance accuracy 0.1g
- 14 Filter paper.

### **Procedure**

#### *Specimen Preparation*

1. Remove the collar of the mould. Measure the internal dimensions of the mould. Weigh the mould with dummy plate to the nearest gram.
2. Apply a little grease on the inside to the mould. Clamp the mould between the base plate and the extension collar and place the assembly on a solid base.
3. Take about 2.5kg of the soil sample, from a thoroughly mixed wet soil, in the mould. Compact the soil at the required dry density using a suitable compacting device.
4. Take a small specimen of the soil in a container for the water content determination.
5. Remove the collar and base plate. Trim the excess soil level with the top of the mould.
6. Clean the outside of the mould and the dummy plate. Find the mass of the soil in the mould.
7. The mould with the sample is now placed over the permeameter. This will have drainage and cap discs properly saturated

### **Test Procedure**

1. Through the top inlet of the constant head reservoir, the specimen is connected.
2. The bottom outlet is opened and a steady flow is established
3. For a particular time interval, the quantity of flow can be collected.
4. Measure the difference of head (h) in levels between the constant head reservoir and the outlet in the base.
5. For the same interval, this is repeated three times.

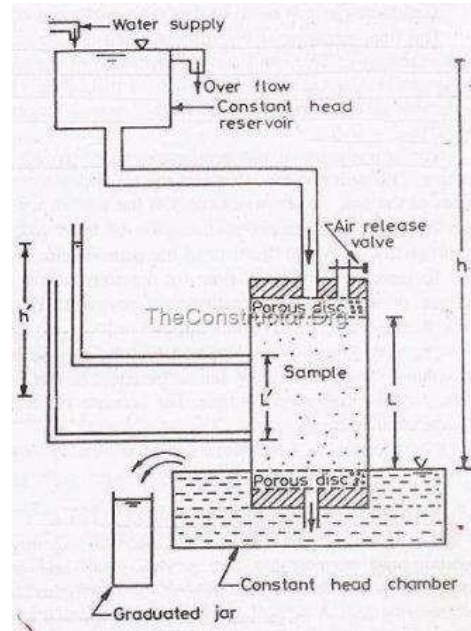


Fig.7.4: Constant Head Permeameter setup

### Observation and Calculations

Initially, the flow is very slow. It later increases and will become constant. The constant head permeability test is best for cohesionless soils.

#### *Observations and Computation in Constant Head Permeameter*

The data that is obtained directly from the tests are:

1. Length (L) in cm
2. Area (A) in cm<sup>2</sup>
3. Constant Head (H) in cm
4. Discharge (Q) in liter

Calculate the coefficient of permeability (k) using the following equation:

$$k = Q \times L / (A \times t \times h) \text{-----}$$

Eq.7.20

where: k = coefficient of permeability,

Q = quantity of water discharged,

L = distance between manometers,

A = cross-sectional area of specimen,

t = total time of discharge

h = head required for flow

### 7.5 FALLING HEAD PERMEABILITY TEST

Falling head permeability test is one of several techniques by which the permeability of soil is determined. It is used to evaluate the permeability of fairly less pervious soil particularly for fine



grained soil sample. Permeability is the measure of the ability of soil to allow water to flow its pores or voids.

A falling head permeameter is shown in Fig.7.5. The soil sample is kept in a vertical cylinder of cross-sectional area  $A$ . A transparent stand pipe of cross sectional area,  $a$ , is attached to the test cylinder. The test cylinder is kept in a container filled with water, the level of which is kept constant by overflows. Before the commencement of the test the soil sample is saturated by allowing the water to flow continuously through the sample from the stand pipe. After saturation is complete, the stand pipe is filled with water up to a height of  $h_0$  and a stop watch is started. Let the initial time be  $t_0$ . The time  $t_1$  when the water level drops from  $h_0$  to  $h_1$  is noted. The hydraulic conductivity  $k$  can be determined on the basis of the drop in head ( $h_0 - h_1$ ) and the elapsed time ( $t_1 - t_0$ ) required for the drop as explained below.

Let  $h$  be the head of water at any time  $t$ . Let the head drop by an amount  $dh$  in time  $dt$ . The quantity of water flowing through the sample in time  $dt$  from Darcy's law is

$$dQ = kiAdt = k \frac{h}{L} Adt \text{-----} \quad \text{Eq.7.21}$$

where,  $i = h/L$  the hydraulic gradient. The quantity of discharge  $dQ$  can be expressed as

$$dQ = -adh \text{-----} \quad \text{Eq.7.22}$$

Since the head decreases as time increases,  $dh$  is a negative quantity in Eq.7.22. & Eq. 7.21 can be equated to Eq. 7.22

$$\frac{-adh}{h} = kAdt/L \text{-----} \quad \text{Eq.7.23}$$

The discharge  $Q$  in time ( $t_1 - t_0$ ) can be obtained by integrating Eq. 2 or 3. Therefore, Eq.4 can be rearranged and integrated as follows

$$-a \int_{h_1}^{h_0} \frac{dh}{h} = \frac{kA}{L} \int_{t_0}^{t_1} dt \text{-----} \quad \text{Eq.7.24}$$

$$\text{Or } a \ln \frac{h_0}{h_1} = \frac{kA}{L} (t_1 - t_0) \text{-----} \quad \text{Eq.7.25}$$

The general expression for  $k$  is

$$K = \frac{aL}{A(t_1 - t_0)} \ln \frac{h_0}{h_1} \text{-----} \quad \text{Eq.7.26}$$

OR

$$K = \frac{2.3aL}{A(t_1 - t_0)} \log_{10} \frac{h_0}{h_1} \text{-----} \quad \text{Eq.7.27}$$

Apparatus

#### 1. Mould Assembly

The mould assembly including drainage base and drainage cap which need to be conform to IS: 11209-1985

#### 2. Compaction Hammer

#### 3. Set of Stand Pipes

Glass stand pipes varying in diameter from 5 to 20 mm, suitably mounted on stand or otherwise fixed on wall

#### 4. Constant Head Tank

Suitable water reservoir capable of supplying water to the permeameter under constant head for constant head test arrangement is required.

#### 5. Miscellaneous Apparatus

For instance, IS sieves, mixing pan, graduated cylinder, meter scale, stop watch, 75micron wire gauge, thermometer, and a source of de-aired water.

#### *Soil Specimen Preparation (Disturbed Soil Sample)*

- Take 2.5-kg soil from a thoroughly mixed air-dried or oven-dried material and evaluate its moisture content.
- Remove the collar of the mould. Measure the internal dimensions of the mould. Weigh the mould with dummy plate to the nearest gram.
- Apply a little grease on the inside to the mould.
- Clamp the mould between the base plate and the extension collar and place the assembly on a solid base.
- Place soil specimen in the mould, and compact it at the required dry density using a suitable compacting device.
- Take a small specimen of the soil in a container for the water content determination.
- Remove the collar and base plate. Trim the excess soil level with the top of the mould.
- Clean the outside of the mould and the dummy plate.
- Find the mass of the soil in the mould.
- The mould with the sample is now placed over the permeameter. This will have drainage and cap discs properly saturated

#### *Undisturbed Soil Sample*

- Trim the specimen in a form of cylinder not larger than 85cm in diameter, and having a height equal to that of the mould.
- Place the specimen over porous disc of the drainage base fixed to the mould.
- Use impervious material like cement slurry to fill the space between mould and the specimen.
- Fix the drainage cap over the top of the mould.

#### **Procedure**

1. Connect the specimen to the selected stand-pipe through the top inlet.
2. Open the bottom outlet and record the time interval required for the water level to fall from a known initial head to a known final head as measured above the center of the outlet.

3. Refill the stand-pipe with water and repeat the test till three successive observations give nearly same time interval; the time intervals being recorded for the drop in head from the same initial to final values, as in the first determination.
4. Alternatively, after selecting the suitable initial and final heads,  $h_1$ , and  $h_2$ , respectively, observe the time intervals for the head to fall from  $h_1$  to  $\sqrt{h_1 h_2}$  and similarly from  $\sqrt{h_1 h_2}$  to  $h_2$ .
5. The time intervals should be the same; otherwise the observation shall be repeated after refilling the stand-pipe.

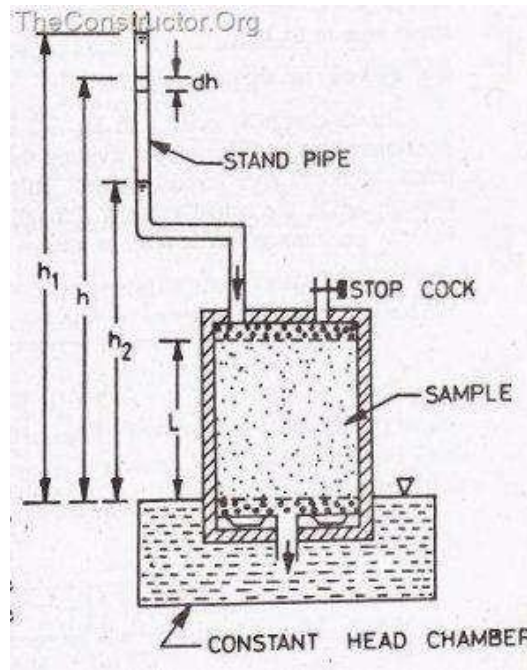


Fig.7.5: Falling Head Permeameter Test Setup

### Data Sheet

The following values shall be recorded in the data sheet of variable head permeability test:

Length of specimen (L)

Diameter of specimen (D)

Volume of specimen (V)

Water content ( $\omega$ )

Diameter of stand pipe (d)

Area of stand pipe (a)

Specific gravity of solids ( $G_s$ )

### Calculations

At temperature  $T$  of water, the permeability  $k_T$  is calculated using the following expression:

$$k_T = \frac{[2.30aL]}{(At)] \log_{10}(h_1/h_2)} \quad \text{-----} \quad \text{Eq.7.28}$$

Where

$h_1$ : initial head

$h_2$ : final head

$t$ : time interval

$a$ : cross-sectional area of the liquid stand pipe

$A$ : cross-sectional area of the specimen

$L$ : length of specimen.

#### Result

The permeability values at temperatures  $T$  and  $27^\circ\text{C}$  are reported as numbers with units as cm/s. The state of the sample is also reported in terms of water content, void ratio and degree of saturation.

**Example 1:** A sand sample of  $35\text{ cm}^2$  cross sectional area and 20 cm long was tested in a constant head permeameter. Under a head of 60 cm, the discharge was 120 ml in 6 min. The dry weight of sand used for the test was 1120 g, and  $G_s = 2.68$ . Determine (a) the hydraulic conductivity in cm/sec, (b) the discharge velocity, and (c) the seepage velocity.

#### Solution

Use Equation,  $k = QL / hAt$

where ,  $Q = 120\text{ ml}$ ,  $t = 6\text{ min}$ ,  $A = 35\text{ cm}^2$  ,  $L = 20\text{ cm}$ , and  $h = 60\text{ cm}$ . Substituting, we have

$$k = 3.174 \times 10^{-3} \text{ cm/sec}$$

$$\text{Discharge velocity, } v = ki = 3.174 \times 10^{-3} \times 60/20 = 9.52 \times 10^{-3} \text{ cm/sec}$$

### 7.6 DIRECT DETERMINATION OF “ $k$ ” OF SOILS IN PLACE BY PUMPING TEST

The most reliable information concerning the permeability of a deposit of coarse grained material below the water table can usually be obtained by conducting pumping tests in the field. Although such tests have their most extensive application in connection with dam foundations, they may also prove advisable on large bridge or building foundation jobs where the water table must be lowered. The arrangement consists of a test well and a series of observation wells. The test well is sunk through the permeable stratum up to the impermeable layer. A well sunk into a water bearing stratum, termed an *aquifer*, and tapping free flowing ground water having a free ground water table under atmospheric pressure, is termed a *gravity* or *unconfined well*. A well sunk into an aquifer where the ground water flow is confined between two impermeable soil layers, and is under pressure greater than atmospheric, is termed as *artesian* or *confined well*. Observation wells are drilled at various distances from the test or pumping well along two straight lines, one oriented approximately in the direction of ground water flow and the other at right angles to it. A minimum of two observation wells and their distances from the test well are needed. These wells are to be provided on one side of the test well in the direction of the ground water flow.

The test consists of pumping out water continuously at a uniform rate from the test well until the

water levels in the test and observation wells remain stationary. When this condition is achieved the water pumped out of the well is equal to the inflow into the well from the surrounding strata. The water levels in the observation wells and the rate of water pumped out of the well would provide the necessary additional data for the determination of  $k$ . As the water from the test well is pumped out, a steady state will be attained when the water pumped out will be equal to the inflow into the well. At this stage the depth of water in the well will remain constant. The drawdown resulting due to pumping is called the *cone of depression*. The maximum drawdown  $DQ$  is in the test well. It decreases with the increase in the distance from the test well. The depression dies out gradually and forms theoretically, a circle around the test well called the *circle of influence*. The radius of this circle is called the *radius of influence* of the depression cone.

### 7.6.1 Equation for $k$ for an Unconfined Aquifer

Figure below gives the arrangement of test and observation wells for an unconfined aquifer. Only two observation wells at radial distances of  $r_1$  and  $r_2$  from the test well are shown. When the inflow of water into the test well is steady, the depths of water in these observation wells are  $h_1$  and  $h_2$  respectively.

Let  $h$  be the depth of water at radial distance  $r$ . The area of the vertical cylindrical surface of radius  $r$  and depth  $h$  through which water flows is given as

$$A = 2\pi rh \text{----- Eq.7.29}$$

The hydraulic gradient is  $i = \frac{dh}{dr}$

As per Darcy's law the rate of inflow into the well when the water levels in the wells remain stationary is

$$q = kiA \text{----- Eq.7.30}$$

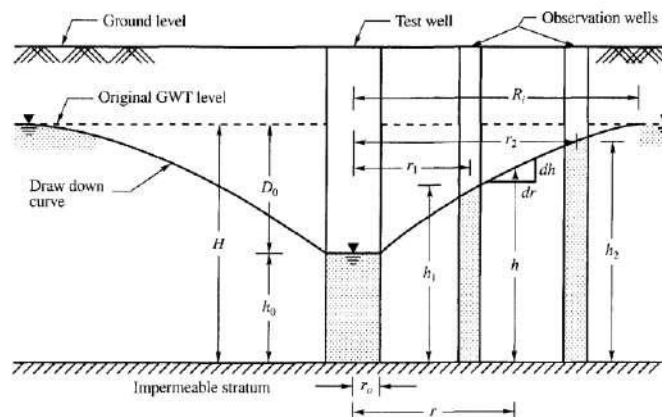


Fig.7.6: Pumping test conducted on Unconfined Aquifer

Substituting for  $A$  and  $i$  the rate of inflow across the cylindrical surface is

$$q = k \frac{dh}{dr} 2\pi rh \text{----- Eq.7.31}$$

Rearranging the terms, we have

$$\frac{dr}{r} = \frac{2\pi kh dh}{q} \text{----- Eq.7.32}$$

The integral of the equation within the boundary limits is

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2k}{q} \int_{h_1}^{h_2} h dh \text{-----} \quad \text{Eq.7.33}$$

The equation for  $k$  after integration and rearranging is

$$k = \frac{q}{2} \frac{\log \frac{r_2}{r_1}}{\frac{\pi(h_2^2 - h_1^2)}{2}} \text{-----} \quad \text{Eq.7.34}$$

Proceeding in the same way as before another equation for  $k$  in terms of  $r_0$ ,  $h_0$  and  $R_i$  can be established as (referring to Fig.7.6)

$$k = \frac{q}{2} \frac{\log \frac{R_i}{r_0}}{\frac{\pi(H^2 - h_0^2)}{2}} \text{-----} \quad \text{Eq.7.35}$$

If we write  $h_0 = (H - D_0)$  in the above Eq.7.35, where  $D_0$  is the depth of maximum drawdown in the test well, we have

$$k = \frac{q}{2} \frac{\log \frac{R_i}{r_0}}{\frac{\pi D_0(2H - D_0)}{2}} \text{-----} \quad \text{Eq.7.36}$$

Now the maximum yield from the well may be written from the above expression as

$$q = \frac{\pi D_0(2H - D_0)k}{2.3} \log \frac{R_i}{r_0} \text{-----} \quad \text{Eq.7.37}$$

### 7.6.2 Equation for $k$ in a Confined Aquifer

Figure 7.7 shows a confined aquifer with the test and observation wells. The water in the observation wells rises above the top of the aquifer due to artesian pressure. When pumping from such an artesian well two cases might arise. They are:

**Case 1.** The water level in the test well might remain above the roof level of the aquifer at steady flow condition.

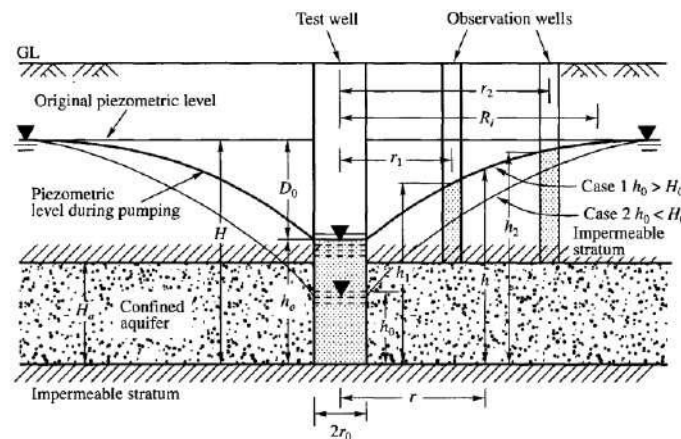


Fig.7.7: Pumping test conducted on Confined Aquifer

**Case 2.** The water level in the test well might fall below the roof level of the aquifer at steady flow condition.

If  $H_0$  is the thickness of the confined aquifer and  $h_0$  is the depth of water in the test well at the steady flow condition Case 1 and Case 2 may be stated as

**Case 1.** When  $h_0 > H_0$  AND **Case 2.** When  $h_0 < H_0$



**Case 1. When  $h_0 > H_0$** 

In this case, the area of a vertical cylindrical surface of any radius  $r$  does not change, since the depth of the water bearing strata is limited to the thickness  $H_0$ . Therefore, the discharge surface area is

$$A = 2\pi r H_0 \text{-----} \quad \text{Eq.7.38}$$

A gain writing  $i = \frac{dh}{dr}$  the flow equation as per Darcy's

$$q = kiA = k \frac{dh}{dr} 2\pi r H_0 \text{-----} \quad \text{Eq.7.39}$$

The integration of the equation after rearranging the terms yields

$$\int_{h_1}^{h_2} dh = \frac{q}{2\pi k H_0} \int_{r_1}^{r_2} \frac{dr}{r} \text{-----} \quad \text{Eq.7.40}$$

$$\text{Or } (h_2 - h_1) = \frac{q}{2\pi k H_0} \ln \frac{r_2}{r_1} \text{-----} \quad \text{Eq.7.41}$$

The equation for  $k$  is

$$k = \frac{q}{2\pi H_0 (h_2 - h_1)} \log \frac{r_2}{r_1} \text{-----} \quad \text{Eq.7.42}$$

**Alternate Equations —**

As before we can write the following equation for determining  $k$

$$k = \frac{q}{2\pi H_0 (h_1 - h_0)} \log \frac{r_1}{r_0} \text{-----} \quad \text{Eq.7.43}$$

$$k = \frac{q}{2\pi H_0 (H - h_0)} \log \frac{R_i}{r_0} \text{-----} \quad \text{Eq.7.44}$$

$$k = \frac{q}{2\pi H_0 D_0} \log \frac{R_i}{r_0} \text{-----} \quad \text{Eq.7.45}$$

**Case 2. When  $h_0 < H_0$** 

Under the condition when  $h_0$  is less than  $H_0$ , the flow pattern close to the well is similar to that of an unconfined aquifer whereas at distances farther from the well the flow is artesian. Muskat (1946) developed an equation to determine the hydraulic conductivity. The equation is

$$k = \frac{2.3q}{2\pi (H H_0 - H_0^2 - h^2)} \log \frac{R_i}{r_0} \text{-----} \quad \text{Eq.7.46}$$

**Example No.1**

A pumping test was carried out for determining the hydraulic conductivity of soil in place. A well of diameter 40 cm was drilled down to an impermeable stratum. The depth of water above the bearing stratum was 8 m. The yield from the well was 4 m<sup>3</sup>/min at a steady drawdown of 4.5 m. Determine the hydraulic conductivity of the soil in m/day if the observed radius of influence was 150m.

Solution

We know that  $k = \frac{2.303 q}{\pi D_0 (2H - D_0) \log \frac{R_i}{r_0}}$

Here  $q = 4 \text{ m}^3/\text{min} = 4 \times 60 \times 24 \text{ m}^3/\text{day}$

$D_0 = 4.5 \text{ m}$ ,  $H = 8 \text{ m}$ ,  $R_i = 150 \text{ m}$ ,  $r_0 = 0.2 \text{ m}$

By substituting above values we get

$k = 234.4 \text{ m/day}$

**Example No.2**

A field pumping test was conducted from an aquifer of sandy soil of 4 m thickness confined between two impervious strata. When equilibrium was established, 90 liters of water was pumped out per hour.

The water elevation in an observation well 3.0 m away from the test well was 2.1 m and another 6.0 m away was 2.7 m from the roof level of the impervious stratum of the aquifer. Find the value of  $k$  of the soil in m/sec.

Solution:

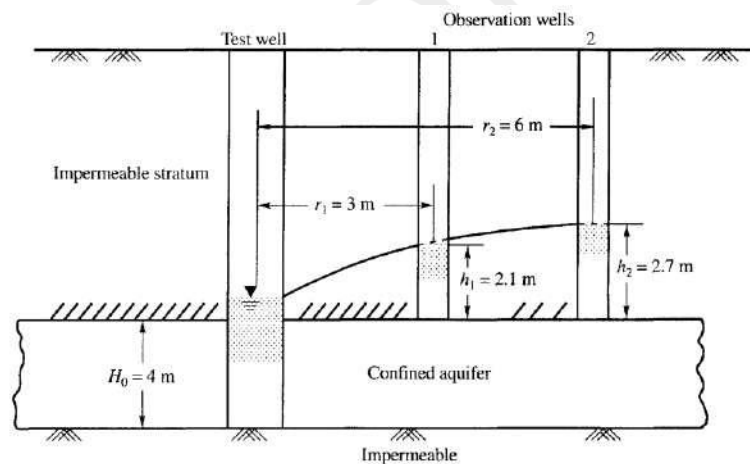
We know that  $k$  for confined aquifer is given as

$$k = \frac{2.3q}{2\pi H_0 (h_2 - h_1)} \log_{10} \frac{r_2}{r_1}$$

$$q = 90 \times 10^3 \text{ cm}^3/\text{hr} = 25 \times 10^{-6} \text{ m}^3/\text{sec}$$

$$H_0 = 4 \text{ m}, r_1 = 3 \text{ m and } r_2 = 6 \text{ m}$$

Substituting the above values we get  $k = 1.148 \times 10^{-6} \text{ m/s}$

**Problem No.1**

Calculate the yield per hour from a well driven into a confined aquifer. The following data are Available, height of original piezometric level from the bed of the aquifer,  $H = 30\text{m}$  thickness of aquifer,  $H_0 = 4\text{m}$ , the depth of water in the well at steady state,  $h_0 = 6 \text{ m}$  hydraulic conductivity of soil =  $0.079 \text{ m/min}$ , radius of well,  $r_0 = 0.25 \text{ m}$ , radius of influence,  $R_i = 150 \text{ m}$

**7.7 PUMPING IN TESTS**

- 1 Open end Test
- 2 Packer Test

**Open-end Tests**

An open end pipe is sunk into the strata and the soil is taken out of the pipe. Clear water having temperature slightly higher than the ground water is added through a metering system. During

this stage flow should maintain gravity flow under constant head. Water may also be allowed to enter the pipe under some pressure head.

The permeability is calculated from the expression as

$$k = \frac{q}{5.5rh} \text{-----} \quad \text{Eq.7.47}$$

Where  $q$  = constant rate of flow

$h$  = Difference head of flow (gravity head and pressure head)

$r$  = radius of the casing

### **Packer Test**

Packers are primarily used in bore holes for testing the permeability of rocks under applied pressures. The apparatus used for the pressure test is comprised of a water pump, a manually adjusted automatic pressure relief valve, pressure gage, a water meter and a packer assembly. The packer assembly consists of a system of piping to which two expandable cylindrical rubber sleeves, called packers, are attached. The packers which are provided by means of sealing a limited section of bore hole for testing should have a length five times the diameter of the hole. They may be of the pneumatically or mechanically expandable type. The former are preferred since they adapt to an oversized hole whereas the latter may not. However, when pneumatic packers are used, the test apparatus must also include an air or water supply connected, through a pressure gage, to the packers by means of a higher pressure hose. The piping of a packer assembly is designed to permit testing of either the portion of the hole between the packers or the portion below the lower packer. The packers are usually set 50, 150 or 300 cm apart. The wider spacing is used for rock which is more uniform. The short spacing is used to test individual joints which may be the cause of high water loss in otherwise tight strata.

Two types of packer methods are used for testing of permeability. They are:

#### **1. Single packer method.**

#### **2. Double packer method.**

The single packer method is useful where the full length of the borehole cannot stand uncased/un-grouted in soft rocks, such as soft sand stone, clay shale or due to the highly fractured and sheared nature of the rocks, or where it is considered necessary to have permeability values side by side with drilling. Where the rocks are sound and the full length of the hole can stand without casing/grouting, the double packer method may be adopted. The disadvantage of the double packer method is that leakage through the lower packer can go unnoticed and lead to overestimation of water loss.

### **Single Packer Method**

The method used for performing water percolation tests in a section of a drilled hole using a single packer is shown in Fig. 3 a. In this method the hole should be drilled to a particular depth desirable for the test. The core barrel should then be removed and the hole cleaned with water. The packer should be fixed at the desired level above the bottom of the hole and the test performed. Water should be pumped into the section under pressure. Each pressure should be maintained until the readings of water intake at intervals of 5 min show a nearly constant reading of water intake for one particular pressure. The constant rate of water intake should be noted.

After performing the test the entire assembly should be removed. The drilling should then proceed for the next test section.

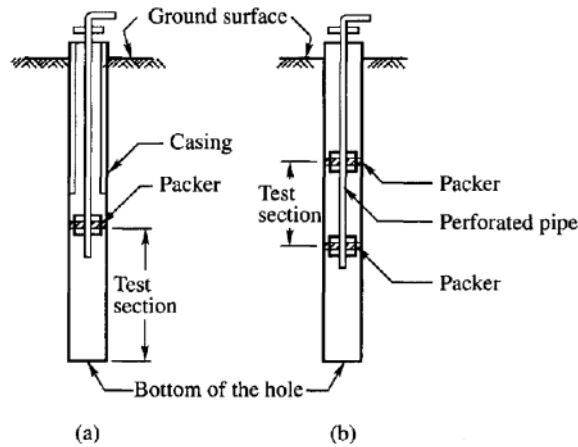


Fig.7.8: Test sections for single and double Packer Method

### Double Packer Method

In this method the hole is first drilled to the final depth and cleaned. The packer assembly may be fixed at any desired test section as shown in Fig. 3 b.

Both packers are then expanded and water under pressure is introduced into the hole between the packers. The tests are conducted as before.

Regardless of which procedure is used, a minimum of three pressures should be used for each section tested. The magnitudes of these pressures are commonly 100, 200 and 300 kPa. (1,2 and 3 kg/cm<sup>2</sup>) above the natural piezometric level. However in no case should the excess pressure be greater than about 20 kPa per meter of soil and rock overburden above the upper packer. The limitation is imposed to insure against possible heavy damage to the foundation.

The formulae used to compute the permeability from pressure test data are (from US Bureau of Reclamation, 1968)

$$k = \frac{q}{2\pi LH} \ln \frac{L}{r_0} \quad \text{for } L \geq 10r_0 \text{ and } \text{-----} \quad \text{Eq.7.48}$$

$$k = \frac{q}{2\pi LH} \sinh^{-1} \frac{L}{2r_0} \quad \text{for } 10r_0 > L \geq r_0 \text{-----} \quad \text{Eq.7.49}$$

where

$k$  = hydraulic conductivity

$q$  = constant rate of flow into the hole

$L$  = length of the test section

$H$  = differential head on the test section

**Example:** A sample in a variable head permeameter is 8 cm in diameter and 10 cm high. The permeability of the sample is estimated to be  $10 \times 10^{-4}$  cm/sec. If it is desired that the head in the stand pipe should fall from 24 cm to 12 cm in 3min, determine the size the stand pipe which should be used.

Sol:

Variable head Permeameter:

Soil sample diameter=8 cm, Height of sample =10 cm

Estimated coefficient of permeability:  $K=10 \times 10^{-4}$  cm/sec

$h_1=24$  cm and  $h_2=12$  cm, time  $t=180$  sec

We know  $K = 2.303 \frac{aL}{At} \log_{10} \left( \frac{h_1}{h_2} \right)$

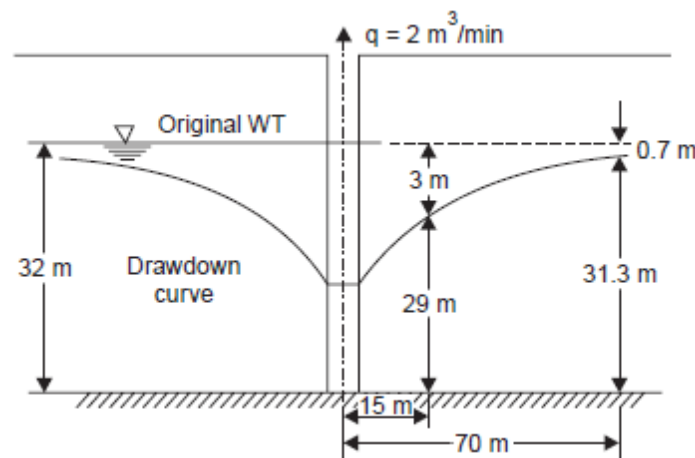
Substituting for various values, we get

$$a = \frac{\pi \times 16 \times 180}{2.303 \times 10^4 (\log_{10} 2)}$$

Hence  $a=1.305 \text{ cm}^2$

Diameter of stand pipe  $d=1.29$  cm

**Example** An unconfined aquifer is known to be 32 m thick below the water table. A constant discharge of 2 cubic metres per minute is pumped out of the aquifer through a tubewell till the water level in the tubewell becomes steady. Two observation wells at distances of 15 m and 70 m from the tubewell show falls of 3 m and 0.7 m respectively from their static water levels. Find the permeability of the aquifer.



Solution:

$$k = \frac{2.3q}{\pi(h_1^2 - h_2^2)} \log_{10} \frac{r_2}{r_1}$$

$$= \frac{2.3 \times 2 \times 100}{\pi(31.3^2 - 29^2) \times 60} \log_{10} \frac{70}{15} = 1.18 \times 10^{-1} \text{ mm/sec}$$

## 8.0 SEEPAGE ANALYSIS

In this class we will discuss about the Laplace equation for determining two-dimensional flow of soil elements.

**The assumptions made in deriving the Laplace equation, the following may be stated as the assumptions of Laplace equation:**

1. The flow is two-dimensional.
2. The flow is steady and laminar.

3. Water and the soil are incompressible.
4. The soil mass is homogeneous and isotropic.
5. The soil is fully saturated and Darcy's law is valid.



Consider a soil element of infinitesimally small size of  $dx$  and  $dz$  in X- and Z-directions, respectively, through which the flow is taking place, shown in Fig. 8.1. Consider unit length of the soil element in the Y-direction.

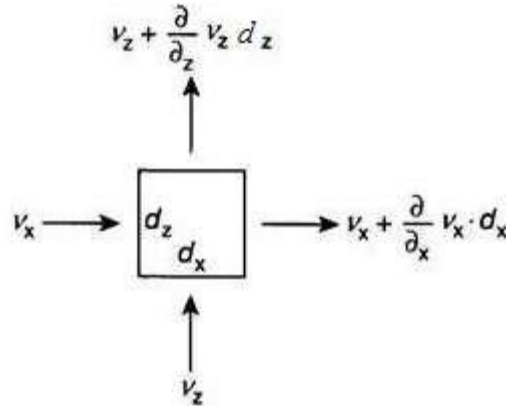


Fig.8.1: Soil element with two dimensional flow

Let  $v_x$  be the velocity of flow at the entry in the X-direction and  $v_z$  the velocity of flow at entry in the Z-direction. Then the velocity at the exit in the X-direction will be

$$v_x + \left(\frac{\partial}{\partial x}\right)(v_x).dx$$

and the velocity at the exit in the Z-direction will be –

$$v_z + \left(\frac{\partial}{\partial z}\right)(v_z).dz$$

Quantity of water entering the element in the X-direction is given by –

$$q_{ix} = v_x.(dz.1) = v_x.dz$$

Quantity of water entering the element in the Z-direction is given by –

$$q_{iz} = v_z.(dx.1) = v_z.dx$$

Similarly, quantity of water leaving the soil element in the X-direction is

$$q_{ox} = \left[v_x + \left(\frac{\partial}{\partial x}\right)(v_x).dx\right] \times (dz.1) = \left[v_x + \left(\frac{\partial}{\partial x}\right)(v_x).dx\right] \times (dz) \text{-----}$$

Eq.8.1

Quantity of water leaving the soil element in the Z-direction is –

$$q_{oz} = \left[v_z + \left(\frac{\partial}{\partial z}\right)(v_z).dz\right] \times (dx.1) = \left[v_z + \left(\frac{\partial}{\partial z}\right)(v_z).dz\right] \times (dx.) \text{----} \quad \text{Eq.8.2}$$

Assuming that the flow is steady and incompressible, the quantity of water entering the soil element is equal to the quantity of water leaving the soil element –

$$\sum q_i = \sum q_o \text{-----} \quad \text{Eq.8.3}$$

$$\equiv q_{ix} + q_{iz} = q_{ox} + q_{oz} \text{-----} \quad \text{Eq.8.4}$$

$$\equiv v_x dz + v_z dx = \left[v_x + \left(\frac{\partial}{\partial x}\right)(v_x).dx\right] \times (dz) + \left[v_z + \left(\frac{\partial}{\partial z}\right)(v_z).dz\right] \times (dx.) \text{-----} \quad \text{Eq.8.5}$$

Rearranging the terms, we have –

$$\left[v_x + \left(\frac{\partial}{\partial x}\right)(v_x).dx\right] \times (dz) - v_x dz + \left[v_z + \left(\frac{\partial}{\partial z}\right)(v_z).dz\right] \times (dx.) - v_z dx = 0 \text{-----}$$

Eq.8.6

—

$$\equiv v \frac{dz}{dx} + \frac{\partial}{\partial x}(v) dx \frac{dz}{dx} - v \frac{dz}{dx} + v \frac{dx}{dz} + \frac{\partial}{\partial z}(v) dz \frac{dx}{dz} - v \frac{dx}{dz} = 0 \text{-----}$$

Eq.8.7

$$\frac{\partial}{\partial x}(v) dx \frac{dz}{dx} + \frac{\partial}{\partial z}(v) dz \frac{dx}{dz} = 0 \text{-----}$$

Eq.8.8

On simplifying, and dividing throughout with  $dz \cdot dx$ , we have

$$\frac{\partial}{\partial x}(v) + \frac{\partial}{\partial z}(v) = 0 \text{-----}$$

Eq.8.9

Let  $h$  be the total head at any point. Then the component of hydraulic gradient in the X-direction will be

$$i_x = -\partial h / \partial x \text{-----}$$

Eq.8.10

The negative sign is to indicate that the head decreases in the direction of flow. Similarly, the component of hydraulic gradient in the Z-direction will be –

$$i_z = -\partial h / \partial z \text{-----}$$

Eq.8.11

By Darcy's law, we know that –

$$v_x = k_x \cdot i_x = -k_x \cdot \partial h / \partial x \text{-----}$$

Eq.8.12

Similarly

$$v_z = k_z \cdot i_z = -k_z \cdot \partial h / \partial z \text{-----}$$

Eq.8.13

Substituting these values in the above equation, we have

$$\frac{\partial}{\partial x}(k_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial z}(k_z \frac{\partial h}{\partial z}) = 0 \text{-----}$$

Eq.8.14

Assuming that the soil mass is homogeneous; permeability will be same throughout in a given direction. Hence

$$-k \frac{\partial}{\partial x}(\frac{\partial h}{\partial x}) - k \frac{\partial}{\partial z}(\frac{\partial h}{\partial z}) = 0 \text{-----}$$

Eq.8.15

$$\equiv k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \text{-----}$$

Eq.8.16

If the soil is isotropic, then  $k_x = k_z = k$ . Then, we have

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \text{-----}$$

Eq.8.17

This is the Laplace equation for two-dimensional flow. It says that the change of gradient in the x-direction plus the change of gradient in the z-direction is zero. The solution of this equation gives a **family of curves meeting at right angles to each other**. One family of these curves represents flow lines and the other equipotential lines. The graphical representation of solution to the Laplace equation which gives two sets of mutually perpendicular curves is known as **Flow net**.

### 8.1 Velocity Potential

Velocity potential is a scalar function of space and time such that its derivative in any direction gives the component of velocity in that direction. Thus –

$$\phi = kh \text{-----}$$

Eq.8.18

$$\partial \phi / \partial x = v_x \text{-----}$$

Eq.8.19

$$\partial\phi = \partial z = v_z \text{-----}$$

Eq.8.20

Substituting these in Laplace Equation, we have

$$\frac{\partial}{\partial x} \left( \frac{\partial\phi}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\partial\phi}{\partial z} \right) = 0 \text{-----}$$

Eq.8.21

$$\text{or } \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \text{-----}$$

Eq.8.22

This is the Laplace equation in terms of velocity potential.

### 8.2 Stream Function

Stream function is a scalar function of space and time such that its derivative in any direction gives the component of velocity in the perpendicular direction clockwise. Thus a positive stream function when derived would give negative velocity and vice versa, as shown in Fig. 8.2. Referring to Fig. 2, we have

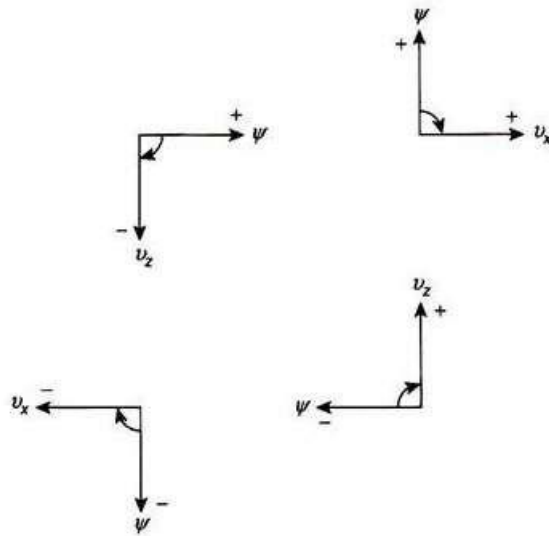


Fig.8.2: Definition of Stream Function

$$\frac{\partial \psi}{\partial x} = -v_z \text{-----}$$

Eq.8.23

$$\frac{\partial \psi}{\partial z} = v_x \text{-----}$$

Eq.8.24

### 8.3 Anisotropic Soil

Soils in nature do possess permeabilities which are different in the horizontal and vertical directions. The permeability in the horizontal direction is greater than in the vertical direction in sedimentary deposits and in most earth embankments. In loess deposits the vertical permeability is greater than the horizontal permeability. The study of flow nets would be of little value if this variation in the permeability is not taken into account. Laplace equation applies for a soil mass where anisotropy exists. This equation may be written in the form

$$\frac{k_z}{k_x} \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \text{-----}$$

Eq.8.25

If we consider a new coordinate variable  $x_c$  measured in the same direction as  $x$  multiplied by a constant, expressed by

$$x_c = x \sqrt{\frac{k_z}{k_x}} \text{-----}$$

Eq.8.26

So the Laplace Equation can be now rewritten as

$$\frac{\partial^2 h}{\partial x_c^2} + \frac{\partial^2 h}{\partial z^2} = 0 \text{-----} \quad \text{Eq.8.27}$$

Now this equation is a *Laplace equation* in the coordinates  $x_c$  and  $z$ . This equation indicates that a cross-section through an anisotropic soil can be transformed to an imaginary section which possesses the same permeability in all directions. The transformation of the section can be effected by multiplying the  $x$ -coordinates by  $\sqrt{\frac{k_z}{k_x}}$  and keeping the  $z$ -coordinates at the natural

scale. The flow net can be sketched on this transformed section. The permeability to be used with the transformed section is

$$k_e = \sqrt{k_z k_x} \text{-----} \quad \text{Eq.8.28}$$

From the transformed section, the rate of seepage can be determined using Eq. 7.1 with exception that  $k_e$  is to be substituted for  $k$  (see Fig. 8.3):

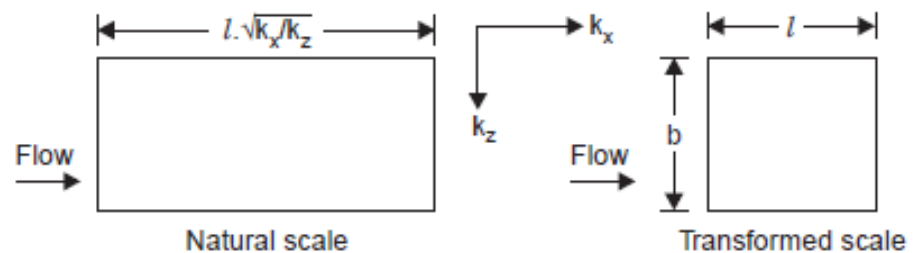


Fig.8.3: Flow in anisotropic soil

## 8.4 FLOW NET

A flow net for an isometric medium is a network of flow lines and equipotential lines intersecting at right angles to each other. The path which a particle of water follows in its course of seepage through a saturated soil mass is called a flow line. Equipotential lines are lines that intersect the flow lines at right angles. At all points along an equipotential line, the water would rise in piezometric tubes to the same elevation known as the piezometric head.

### 8.4.1 FLOW NET CONSTRUCTION

#### Properties of a Flow Net

The properties of a flow net can be expressed as given below:

1. Flow and equipotential lines are smooth curves.
2. Flow lines and equipotential lines meet at right angles to each other.
3. No two flow lines cross each other.
4. Two flow or equipotential lines can not start from the same point.

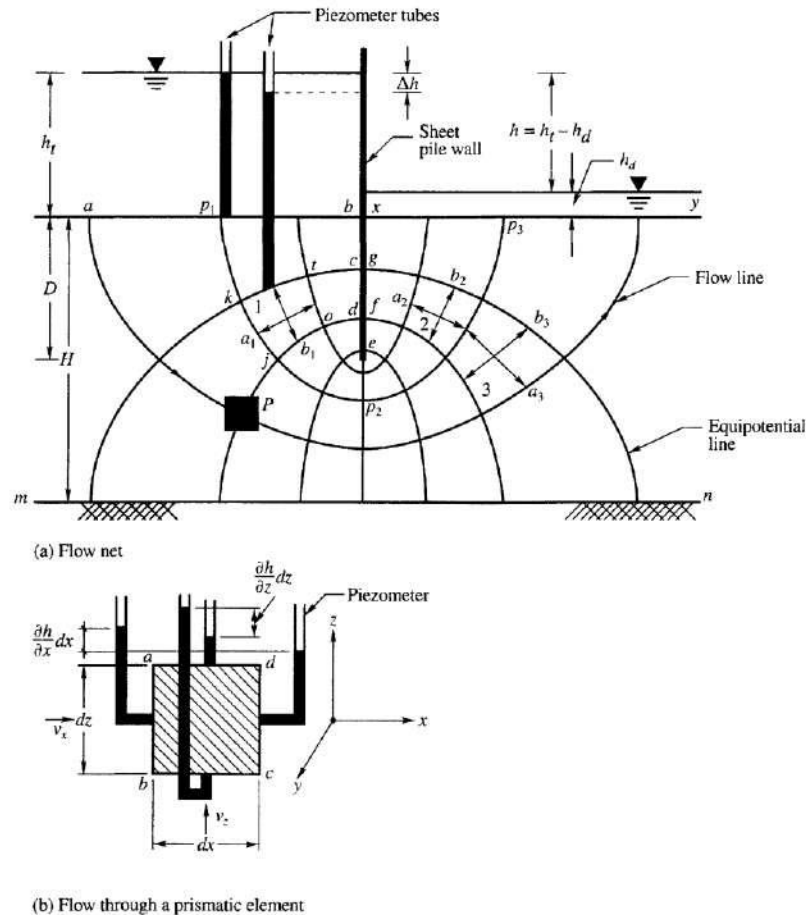


Fig. 8.4: Flow through a homogeneous stratum of soil

#### 8.4.2 Boundary Conditions

Flow of water through earth masses is in general three dimensional. Since the analysis of three-dimensional flow is too complicated, the flow problems are solved on the assumption that the flow is two-dimensional. All flow lines in such a case are parallel to the plane of the figure, and the condition is therefore known as two-dimensional flow. All flow studies dealt with herein are for the steady state case. The expression for boundary conditions consists of statements of head or flow conditions at all boundary points. The boundary conditions are generally four in number though there are only three in some cases. The boundary conditions for the case shown in Fig. 8.4 are as follows:

1. Line  $ab$  is a boundary equipotential line along which the head is  $h_f$
2. The line along the sheet pile wall is a flow boundary
3. The line  $xy$  is a boundary equipotential line along which the head is equal to  $h_d$
4. The line  $mn$  is a flow boundary (at depth  $H$  below bed level).

If we consider any flow line, say,  $p_1, p_2, p_3$  in Fig.8.4, the potential head at  $p_1$  is  $h_1$  and at  $p_3$  is  $h_d$ . The total head lost as the water flows along the line is  $h$  which is the difference between the upstream and downstream heads of water. The head lost as the water flows from  $p_1$  to

equipotential line  $k$  is  $\Delta h$  which the difference between the heads is shown by the piezometers. This loss of head  $\Delta h$  is a fraction of the total head lost.

### 8.5 FLOW NET CONSTRUCTION

Flow nets are constructed in such a way as to keep the ratio of the sides of each block bounded by two flow lines and two equipotential lines a constant. If all the sides of one such block are equal, then the flow net must consist of **squares**. The square block referred to here does not constitute a square according to the strict meaning of the word; it only means that the average width of the square blocks is equal. For example, in Fig.8.4, the width  $a_1$  of block 1 is equal to its length  $b_1$ . The area bounded by any two neighboring flow lines is called a **flow channel**. If the flow net is constructed in such a way that the ratio  $a/b$  remains the same for all blocks, then it can be shown that there is the **same quantity of seepage in each flow channel**. In order to show this consider two blocks 1 and 2 in one flow channel and another block 3 in another flow channel as shown in Figure. Block 3 is chosen in such a way that it lies within the same equipotential lines that bound the block 2. Darcy's law for the discharge through any block such as 1 per unit length of the section may be written as

$$\Delta q = k i a = k \frac{\Delta h}{b} a = k \Delta h \left( \frac{a}{b} \right) \text{-----} \quad \text{Eq.8.29}$$

where  $\Delta h$  represents the head loss in crossing the block. The expressions in this form for each of the three blocks under consideration are

$$\Delta q_1 = k \Delta h \frac{a_1}{b_1} \text{ and } \Delta q_2 = k \Delta h \frac{a_2}{b_2} \text{-----} \quad \text{Eq.8.30}$$

In the above equation the value of hydraulic conductivity  $k$  remains the same for all the blocks. If the blocks are all squares then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = 1$$

Since blocks 1 and 2 are in the same flow channel, we have  $\Delta q_1 = \Delta q_2$ . Since blocks 2 and 3 are within the same equipotential lines we have  $\Delta h_1 = \Delta h_2$ . If these equations are inserted we obtain the following relationship:

$$\Delta q_1 = \Delta q_2 \text{ and } \Delta h_1 = \Delta h_2$$

This proves that the same quantity flows through each block and there is the same head drop in crossing each block if all the blocks are squares or possess the same ratio  $a/b$ . Flow nets are constructed by keeping the ratio  $a/b$  the same in all figures. Square flow nets are generally used in practice as this is easier to construct.

There are many methods that are in use for the construction of flow nets. Some of the important methods are

1. Analytical method,
2. Electrical analog method,
3. Scaled model method,
4. Graphical method.

The analytical method, based on the Laplace equation although rigorously precise, is not universally applicable in all cases because of the complexity of the problem involved. The



mathematics involved even in some elementary cases is beyond the comprehension of many design engineers. Although this approach is sometimes useful in the checking of other methods, it is largely of academic interest.

The electrical analogy method has been extensively made use of in many important for design problems. However, in most of the cases in the field of soil mechanics where the estimation of seepage flows and pressures are generally required, a more simple method such as the graphical method is preferred.

Scaled models are very useful to solve seepage flow problems. Soil models can be constructed to depict flow of water below concrete dams or through earth dams. These models are very useful to demonstrate the fundamentals of fluid flow, but their use in other respects is limited because of the large amount of time and effort required to construct such models.

The graphical method developed by Forchheimer (1930) has been found to be very useful in solving complicated flow problems. A. Casagrande (1937) improved this method by incorporating many suggestions. The main drawback of this method is that a good deal of practice and aptitude are essential to produce a satisfactory flow net. In spite of these drawbacks, the graphical method is quite popular among engineers.

### **8.5.1 Graphical Method**

The usual procedure for obtaining flow nets is a graphical, trial sketching method, sometimes called the Forchheimer Solution. This method of obtaining flow nets is the quickest and the most practical of all the available methods. A. Casagrande (1937) has offered many suggestions to the beginner who is interested in flow net construction. Some of his suggestions are summarized below:

1. Study carefully the flow net pattern of well-constructed flow nets.
2. Try to reproduce the same flow nets without seeing them.
3. As a first trial, use not more than four to five flow channels. Too many flow channels would confuse the issue.
4. Follow the principle of 'whole to part', i.e., one has to watch the appearance of the entire flow net and when once the whole net is found approximately correct, finishing touches can be given to the details.
5. All flow and equipotential lines should be smooth and there should not be any sharp transitions between straight and curved lines.

The above suggestions, though quite useful for drawing flow nets, are not sufficient for a beginner. In order to overcome this problem, Taylor (1948) proposed a procedure known as the procedure by explicit trials. Some of the salient features of this procedure are given below:

1. As a first step in the explicit trial method, one trial flow line or one trial equipotential line is sketched adjacent to a boundary flow line or boundary equipotential.
2. After choosing the first trial line (say it is a flow line), the flow path between the line and the boundary flow line is divided into a number of squares by drawing equipotential lines. These

equipotential lines are extended to meet the bottom flow line at right angles keeping in view that the lines drawn should be smooth without any abrupt transitions.

3. The remaining flow lines are next drawn, adhering rigorously to square figures.

4. If the first trial is chosen property, the net drawn satisfies all the necessary conditions. Otherwise, the last drawn flow line will cross the bottom boundary flow line, indicating that the trial line chosen is incorrect and needs modification.

5. In such a case, a second trial line should be chosen and the procedure repeated.

A typical example of a flow net under a sheet pile wall is given in Fig. 1. It should be understood that the number of flow channels will be an integer only by chance. That means, the bottom flow line sketched might not produce full squares with the bottom boundary flow line. In such a case the bottom flow channel will be a fraction of a full flow channel. It should also be noted that the figure formed by the first sketched flow line with the last equipotential line in the region is of irregular form. This figure is called a singular square. The basic requirement for such squares, as for all the other squares, is that continuous sub-division of the figures give an approach to true squares. Such singular squares are formed at the tips of sheet pile walls also. Squares must be thought of as valid only where the Laplace equation applies. The Laplace equation applies to soils which are homogeneous and isotropic. When the soil is anisotropic, the flow net should be sketched as before on the transformed section. The transformed section can be obtained from the natural section explained earlier.

## 8.6 DETERMINATION OF QUANTITY OF SEEPAGE

Flow nets are useful for determining the quantity of seepage through a section. The quantity of seepage  $q$  is calculated per unit length of the section. The flow through any square can be written as

$$\Delta q = k \Delta h \text{-----} \quad \text{Eq.8.31}$$

Let the number of flow channel and equipotential drops in a section be  $N_f$  and  $N_d$ , respectively. Since all drops are equal, we can write

$$\Delta h = \frac{h}{N_d} \text{-----} \quad \text{Eq.8.32}$$

Since the discharge in each flow channel is the same we can write,

$$q = N_f \Delta q \text{-----} \quad \text{Eq.8.33}$$

Substituting for  $\Delta q$  and  $\Delta h$  we have

$$q = kh \frac{N_f}{N_d} \text{-----} \quad \text{Eq.8.34}$$

This Eq. can also be used to compute the seepage through anisotropic sections by writing  $k_e$  in place of  $k$ , where  $k_e$  is equal to  $\sqrt{k_x k_z}$ , where  $k_x$  and  $k_z$  are the hydraulic conductivities in the  $x$  and  $z$  directions, respectively. The validity of this relationship can be proved as follows. Consider a figure bounded by flow and equipotential lines in which the flow is parallel to the  $x$  direction. In Figure 8.5 the figure in question is drawn to a transformed scale in (b) and the same to the natural scale in (a). In Fig. 8.5 (b) the permeability has the effective value  $k_e$  in both the  $x$  and  $z$  directions and the flow through the square according to

$$\Delta q = k_e \frac{\Delta h a}{a} = k_e \Delta h \text{---} \quad \text{Eq.8.35}$$

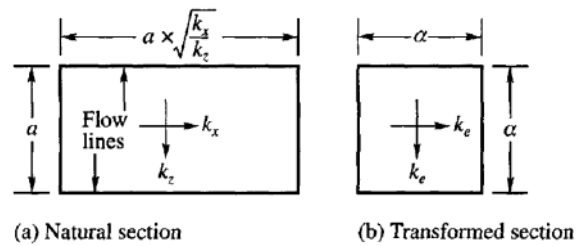


Fig.8.5: Flow through anisotropic soil

In Fig. 8.5 (a) the hydraulic conductivity  $k_x$  in the horizontal section must apply because the flow is horizontal and the sketch is to the natural scale. The flow equation is, therefore,

$$\Delta q = k_x i A = k_x \frac{\Delta h a}{a \sqrt{\frac{k_x}{k_z}}} \quad \text{Eq.8.36}$$

## 8.7 DETERMINATION OF SEEPAGE PRESSURE

Flow nets are useful in the determination of the seepage pressure at any point along the flow path.

Consider the cubical element 1 in Fig.8.4 (a) with all the sides equal to  $a$ . Let  $h_1$  be the piezometric head acting on the face  $kt$  and  $h_2$  on face  $jo$ .

$$\text{The total force on face } kt = P_1 = a^2 \gamma_w h_1 \quad \text{Eq.8.37}$$

$$\text{The total force on face } jo = P_2 = a^2 \gamma_w h_2 \quad \text{Eq.8.38}$$

The differential force acting on the element is

$$P_1 - P_2 = a^2 \gamma_w (h_1 - h_2) \quad \text{Eq.8.39}$$

Since  $(h_1 - h_2)$  is the head drop  $\Delta h$ , we can write

$$P = a^3 \gamma_w \frac{\Delta h}{a} = a^2 \gamma_w \Delta h = a^3 \gamma_w i \quad \text{Eq.8.40}$$

$$P = \frac{a^3}{3} \gamma_w \quad \text{where } a^3 \text{ is volume}$$

Where,  $a^3$  is the volume of the element. The force per unit volume of the element is, therefore

$$P_s = \gamma_w i \quad \text{Eq.8.41}$$

This force exerts a drag on the element known as the *seepage pressure*. It has the dimension of unit weight, and at any point its line of action is tangent to the flow line. The seepage pressure is a very important factor in the stability analysis of earth slopes. If the line of action of the seepage force acts in the vertical direction upward as on an element adjacent to point x, in Fig.8.6 (a), the force that is acting downward to keep the element stable is the buoyant unit weight of the element. When these two forces balance, the soil will just be at the point of being lifted up, and there will be effectively no grain-to-grain pressures.

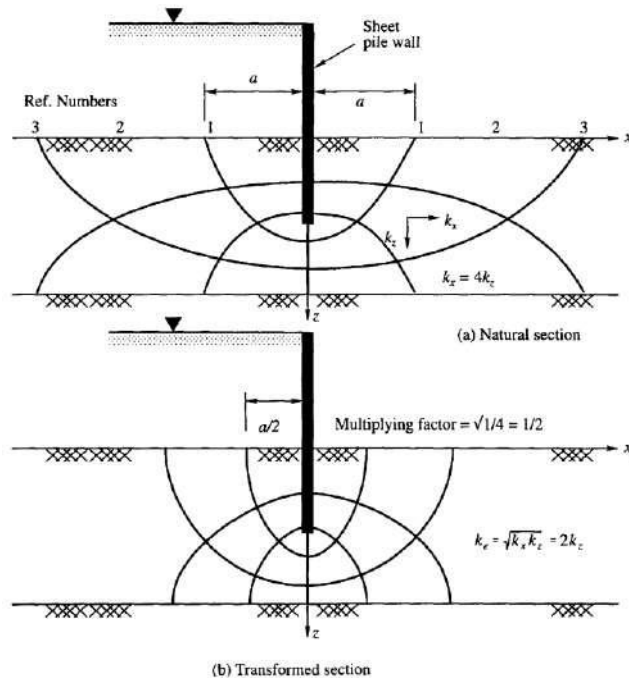


Fig.8.6: Flow-net in anisotropic soil

### 8.8 DETERMINATION OF UPLIFT PRESSURES

Water that seeps below masonry dams or weirs founded on permeable soils exerts pressures on the bases of structures. These pressures are called *uplift pressures*. Uplift pressures reduce the effective weight of the structure and thereby cause instability. It is therefore very essential to determine the uplift pressures on the base of dams or weirs accurately. Accurate flow nets should be constructed in cases where uplift pressures are required to be determined. The method of determining the uplift pressures can be explained as follows.

Consider a concrete dam as shown in Figure 8.7 a founded on a permeable foundation at a depth  $D$  below the ground surface. The thickness of the permeable strata is  $H$ . The depth of water on the upstream side is  $h_1$  and on the downstream side is zero. Water flows from the upstream to the downstream side. It is necessary to determine the uplift pressure on the base of the dam by means of flow nets as shown in the figure.

The difference in head between the upstream and downstream water levels is  $h_f$ . Let the number of equipotential drops be  $N_d$ . The head lost per drop be  $\Delta h = \frac{h_f}{N_d}$ . As the water flows along the

side and base of the dam, there will be equal drops of head between the equipotential lines that meet the dam as shown in the figure. A piezometer tube at point  $a$  (coinciding with the corner of the dam in the figure) gives a pressure head  $h_a$ . Now the uplift pressure at point  $a$  may be expressed as

$$u_a = h_a \gamma_w = (h_f + D - \Delta h) \gamma_w \text{-----} \quad \text{Eq.8.42}$$

Similarly, the uplift pressure at any other point, say  $e$  (see the figure), may be estimated from the expression

$$u_e = (h_l + D - n_d \Delta h) \gamma_w \text{-----}$$

Eq.8.43

where,  $n_d$  = the number of equipotential drops to the point  $e$ .

Fig. 8.7.b shows the distribution of uplift pressure on the base of the dam.

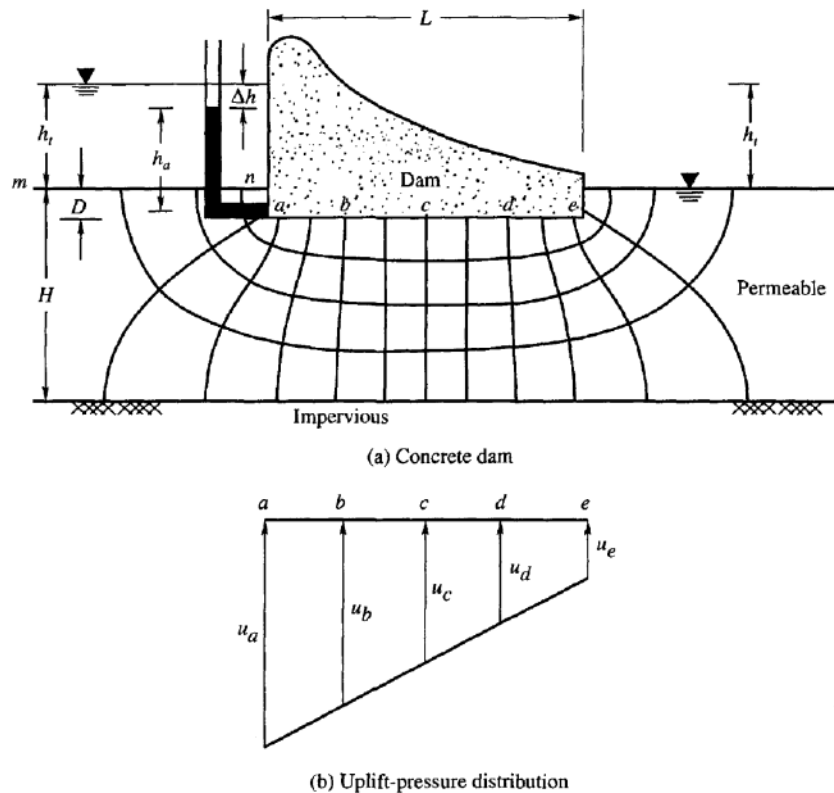
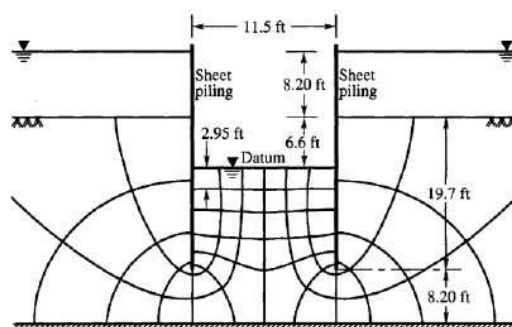


Fig.8.7: Uplift pressure on the base of a concrete dam

### Example 1

Two lines of sheet piles were driven in a river bed as shown in Fig. The depth of water over the river bed is 8.20 ft. The trench level within the sheet piles is 6.6 ft below the river bed. The water level within the sheet piles is kept at trench level by resorting to pumping. If a quantity of water flowing into the trench from outside is 3.23 ft<sup>3</sup>/hour per foot length of sheet pile, what is the hydraulic conductivity of the sand? What is the hydraulic gradient immediately below the trench bed?



**Solution:**

Fig. gives the flow net and other details. The differential head between the bottom of trench and the water level in the river is 14.8 ft.

Number of channels = 6

Number of equipotential drops = 10

Discharge

$$q = kh \frac{N_f}{N_d}$$

From which,  $k = 1 \times 10^{-4}$  ft/sec

The distance between the last two equipotentials given is 2.95 ft. The calculated hydraulic gradient is

$$i = \frac{\Delta h}{\Delta s} = 0.50$$

**8.9 SEEPAGE FLOW THROUGH HOMOGENEOUS EARTH DAMS**

In almost all problems concerning seepage beneath a sheet pile wall or through the foundation of a concrete dam all boundary conditions are known. However, in the case of seepage through an earth dam the upper boundary or the uppermost flow line is not known. This upper boundary is a free water surface and will be referred to as the *line of seepage or phreatic line*. The seepage line may therefore be defined as the line above which there is no hydrostatic pressure and below which there is hydrostatic pressure. In the design of all earth dams, the following factors are very important.

1. The seepage line should not cut the downstream slope.
2. The seepage loss through the dam should be the minimum possible.

The two important problems that are required to be studied in the design of earth dams are:

1. The prediction of the position of the line of seepage in the cross-section.
2. The computation of the seepage loss.

If the line of seepage is allowed to intersect the downstream face much above the toe, more or less serious sloughing may take place and ultimate failure may result. This mishap can be prevented by providing suitable drainage arrangements on the downstream side of the dam. The section of an earth dam may be homogeneous or non-homogeneous. A homogeneous dam contains the same material over the whole section and only one coefficient of permeability may be assumed to hold for the entire section. In the non homogeneous or the composite section, two or more permeability coefficients may have to be used according to the materials used in the section. When a number of soils of different permeabilities occur in a cross-section, the prediction of the position of the line of seepage and the computation of the seepage loss become quite complicated.

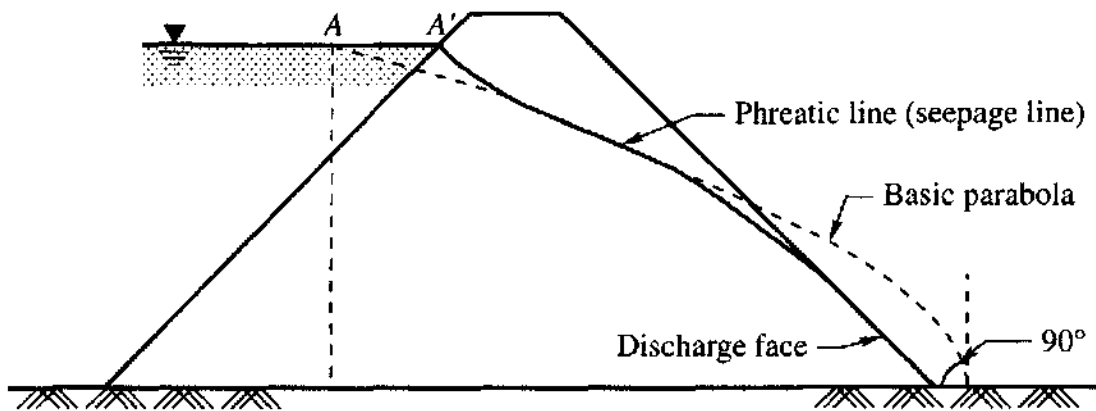


Fig.8.8: Basic parabola and the phreatic line for a homogeneous earth dam

It has been noticed from experiments on homogeneous earth dam models that the line of seepage assumes more or less the shape of a parabola as illustrated in Fig. 8.8. In some sections a little divergence from a regular parabola is required at the surfaces of entry and discharge of the line of seepage. In some ideal sections where conditions are favorable the entire seepage line may be considered as a parabola. When the entire seepage line is a parabola, all the other flow lines will be confocal parabolas. The equipotential lines for this ideal case will be conjugate confocal parabolas as shown in Fig.1. As a first step it is necessary to study the ideal case where the entire flow net consists of conjugate confocal parabolas.

### 8.9.1 Flow net consisting of conjugate confocal parabolas

As a prelude to the study of an ideal flow net comprising of parabolas as flow and equipotential line, it is necessary to understand the properties of a single parabola. The parabola  $ACV$  illustrated in Fig.8.9, is defined as the curve whose every point is equidistant from a point  $F$  called *the focus* and a line  $DG$  called the *directrix*. If we consider any point, say,  $A$ , on the curve, we can write  $FA = AG$ , where the line  $AG$  is normal to the directrix. If  $F$  is the origin of coordinates, and the coordinates of point  $A$  are  $(x, y)$ , we can write

$$AF = \sqrt{x^2 + y^2} = AG = x + y_0 \text{-----} \quad \text{Eq.8.44}$$

$$\text{Or } x = \frac{y^2 - y_0^2}{2y_0} \text{-----} \quad \text{Eq.8.45}$$

where,  $y_0 = FD$

Eq.8.45 is the equation of the basic parabola. If the parabola intersects the  $y$ -axis at  $C$ , we can write

$$FC = CE = y_0$$

Similarly for the vertex point  $V$ , the focal distance  $a_0$  is

$$FV = VD = a_0 = \frac{y_0}{2}$$

Figure 8.9 illustrates the ideal flow net consisting of conjugate confocal parabolas. All the parabolas have a common focus  $F$ .

The boundary lines of such an ideal flow net are:

- 1 . The upstream face  $AB$ , an equipotential line, is a parabola.



2. The downstream discharge face  $FV$ , an equipotential line, is horizontal.
3.  $ACV$ , the phreatic line, is a parabola.
4.  $BF$ , the bottom flow line, is horizontal.

The known boundary conditions are only three in number. They are, the two equipotential lines  $AB$  and  $FV$ , and the bottom flow line  $BF$ . The top flow line  $ACV$  is the one that is unknown. The theoretical investigation of Kozeny (1931) revealed that the flow net for such an ideal condition mentioned above with a horizontal discharge face  $FV$  consists of two families of confocal parabolas with a common focus  $F$ . Since the conjugate confocal parabolas should intersect at right angles to each other, all the parabolas crossing the vertical line  $FC$  should have their intersection points lie on this line.

Since the seepage line is a line of atmospheric pressure only, the only type of head that can exist along it is the elevation head. Therefore, there must be constant drops in elevation between the points at which successive equipotentials meet the top flow line, as shown in Fig.1.

In all seepage problems connected with flow through earth dams, the focus  $F$  of the basic parabola is assumed to lie at the intersection of the downstream discharge face  $FV$  and the bottom flow line  $BF$  as shown in Fig. 8.9. The point  $F$  is therefore known. The point  $A$ , which is the intersection point of the top flow line of the basic parabola and the upstream water level, is also supposed to be known. When the point  $A$  is known, its coordinates  $(d, K)$  with respect to the origin  $F$  can be determined. With these two known points, the basic parabola can be constructed as explained below. We may write

$$AF = AG = \sqrt{d^2 + h^2} \text{-----} \quad \text{Eq. 8.46}$$

$$FD = 2a_0 = \sqrt{d^2 + h^2} - d \text{-----} \quad \text{Eq. 8.47}$$

$$\text{Or } a_0 = \frac{1}{2} \sqrt{d^2 + h^2} - d \text{-----} \quad \text{Eq. 8.48}$$

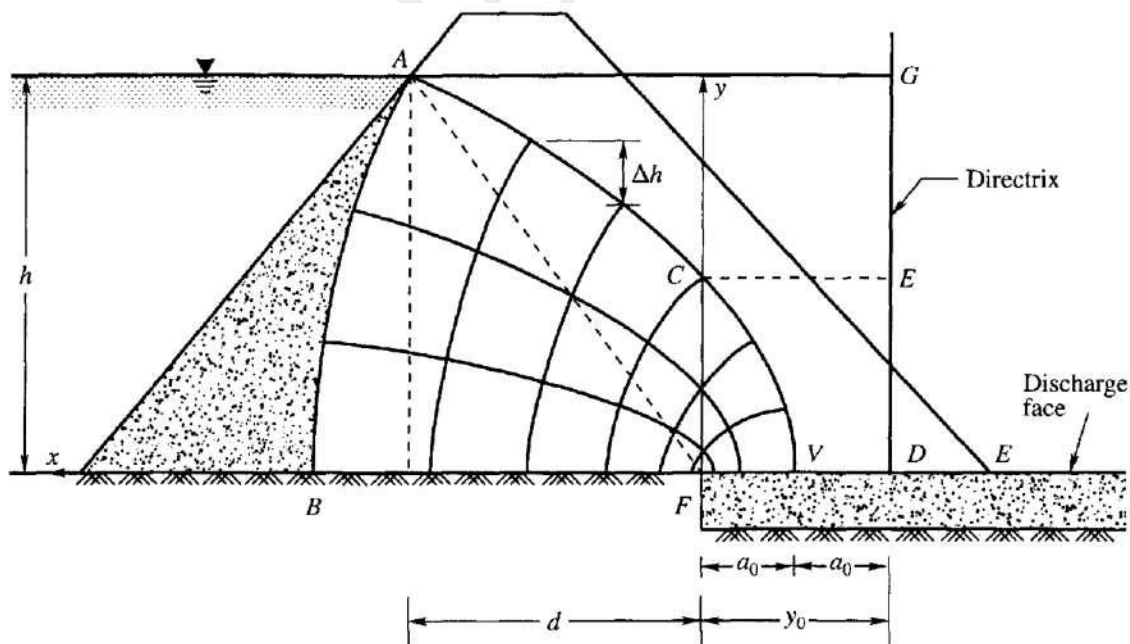


Fig.8.9: Ideal flow net consisting of conjugate confocal parabolas

### 8.9.2 Seepage Loss Through the Dam

The seepage flow  $q$  across any section can be expressed according to Darcy's law as

$$q = kiA$$

Considering the section  $FC$  in Fig. 8.9, where the sectional area  $A$  is equal to  $y_0$ , the hydraulic gradient can be determined analytically as follows:

From Eq. (2), the equation of the parabola can be expressed as

$$y = \sqrt{2xy_0 + y_0^2} \text{-----} \quad \text{Eq. 8.49}$$

The hydraulic gradient  $i$  at any point on the seepage line in Fig. 8.9 can be expressed as

$$\frac{dy}{dx} = \frac{y_0}{\sqrt{2xy_0 + y_0^2}} \text{-----} \quad \text{Eq.8.50}$$

$$\frac{dy}{dx} = \frac{y_0}{\sqrt{2xy_0 + y_0^2}}$$

For the point  $C$  which has coordinates  $(0, y_0)$ , the hydraulic gradient from Eq. (8.50) is

$$\frac{dy}{dx} = \frac{y_0}{\sqrt{y_0^2}} = 1 \text{-----} \quad \text{Eq.8.51}$$

$$\frac{dy}{dx} = \frac{y_0}{\sqrt{y_0^2}}$$

Therefore, the seepage quantity across section  $FC$  is

$$q = k \frac{dy}{dx} y = ky \text{-----} \quad \text{Eq.8.52}$$

### 8.9.3 Seepage Through Homogeneous and Isotropic Earth Dams

#### Types of Entry and Exit of Seepage lines

The flow net consisting of conjugate confocal parabolas is an ideal case which is not generally met in practice. Though the top flow line resembles a parabola for most of its length, the departure from the basic parabola takes place at the faces of entry and discharge of the flow line. The departure from the basic parabola depends upon the conditions prevailing at the points of entrance and discharge of the flow line as illustrated in Fig. 8.10 from (a) to (e).

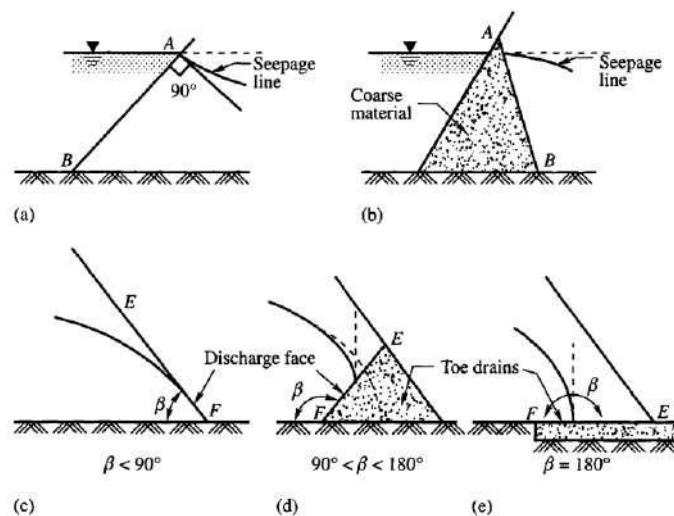


Fig.8.10 Types of entry and exit of seepage lines

The seepage line should be normal to the equipotential line at the point of entry as shown in Fig. 8.10 (a). However, this condition is violated in Fig. 8.10 (b), where the angle made by the upstream

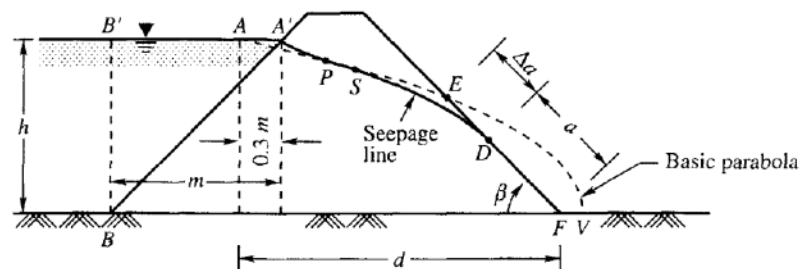
face  $AB$  with the horizontal is less than  $90^\circ$ . It can be assumed in this case the coarse material used to support the face  $AB$  is highly permeable and does not offer any resistance for

flow. In such cases  $AB$  taken as the upstream equipotential line. The top flow line cannot therefore be normal to the equipotential line. However, this line possesses zero gradient and velocity at the point of entry. This zero condition relieves the apparent inconsistency of deviation from a normal intersection. The conditions prevailing at the downstream toe of the dam affect the type of exit of the flow line at the discharge face. In Fig. 8.10 (c) the material at the toe is the same as in the other parts of the dam whereas in (d) and (e) rock toe drains are provided. This variation in the soil condition at the toe affects the exit pattern of the flow line. The flow line will meet the discharge face  $FE$  tangentially in 8.10 (c). This has to be so because the particles of water as they emerge from the pores at the discharge face have to conform as nearly as possible to the direction of gravity. But in cases where rock toe drains are provided, the top flow line becomes tangential to the vertical line drawn at the point of exit on the discharge face as shown in (d) and (e) of Fig. 8.10.

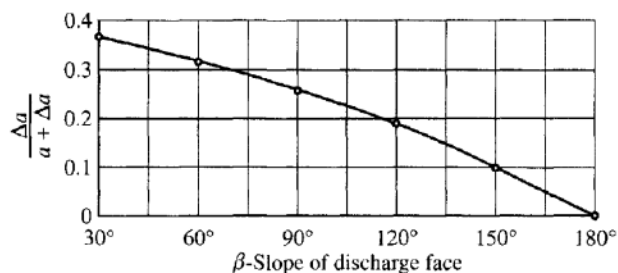
### 8.10 METHOD OF LOCATING SEEPAGE LINE

The general method of locating the seepage line in any homogeneous dam resting on an impervious foundation may be explained with reference to Fig. 8.11 (a).

As explained earlier, the focus  $F$  of the basic parabola is taken as the intersection point of the bottom flow line  $BF$  and the discharge face  $EF$ . In this case the focus coincides with the toe of the dam. One more point is required to construct the basic parabola. Analysis of the location of seepage lines by A. Casagrande has revealed that the basic parabola with focus  $F$  intersects the upstream water surface at  $A$  such that  $AA' = 0.3m$ , where  $m$  is the projected length of the upstream equipotential line  $A'B$  on the water surface. Point  $A$  is called the corrected entrance point.



(a)



(b)

Fig.8.11: Construction of seepage line

The parabola *APSV* may now be constructed as per Eq. (2). The divergence of the seepage line from the basic parabola is shown as *AT1* and *SD* in Fig. 8.11 (a). For dams with flat slopes, the divergences may be sketched by eye keeping in view the boundary requirements. The error involved in sketching by eye, the divergence on the downstream side, might be considerable if the slopes are steeper. Procedures have therefore been developed to sketch the downstream divergence as explained below. As shown in Fig. 8.11 (a), *E* is the point at which the basic parabola intersects the discharge face. Let the distance *ED* be designated as *Aa* and the distance *DF* as *a*. The values of  $\Delta a$  and  $a + \Delta a$ , vary with the angle,  $\beta$ , made by the discharge face with the horizontal measured clockwise. The angle may vary from  $30^\circ$  to  $180^\circ$ . The discharge face is horizontal as shown in Fig. 8.10(e). Casagrande (1937) determined the ratios of  $\frac{\Delta a}{(a + \Delta a)}$  for a

number of discharge slopes varying from  $30^\circ$  to  $180^\circ$  and the relationship is shown in a graphical form Fig. 8.11 (b). The distance  $(a + \Delta a)$  can be determined by constructing the basic parabola with *F* as the focus. With the known  $(a + \Delta a)$  and the discharge face angle  $\beta$ ,  $\Delta a$  can be determined from Fig. 8.11 (b). The point *D* may therefore be marked out at a distance of  $\Delta a$  from *E*. With the point *D* known, the divergence *DS* may be sketched by eye.

It should be noted that the discharge length *a*, is neither an equipotential nor a flow line, since it is at atmospheric pressure. It is a boundary along which the head at any point is equal to the elevation.

#### 8.10.1 Analytical Solutions for Determining *a* and *q*

Casagrande (1937) proposed the following equation for determining '*a*' for  $\beta < 30^\circ$

$$a = \frac{d}{\cos\beta} - \sqrt{\frac{d^2}{\cos^2\beta} - \frac{h^2}{\sin^2\beta}} \text{-----} \text{Eq. 8.53}$$

L. Casagrande (1932) gave the following equation for '*a*' when  $\beta$  lies between  $30^\circ$  and  $90^\circ$ .

$$a = \sqrt{h^2 + d^2} - \sqrt{d^2 - h^2 \cot^2\beta} \text{-----} \text{Eq. 8.54}$$

The discharge *q* per unit length through any cross-section of the dam may be expressed as follows:

$$\text{For } \beta < 30^\circ, q = k \sin\beta \cos\beta \text{-----} \text{Eq. 8.55}$$

$$\text{For } 30^\circ < \beta < 90^\circ, q = k \sin^2\beta \text{-----} \text{Eq. 8.56}$$

### 8.11 PIPING FAILURE

Piping failures caused by heave can be expected to occur on the downstream side of a hydraulic structure **when the uplift forces of seepage exceed the downward forces due to the submerged weight of the soil.**

The mechanics of failure due to seepage was first presented by Terzaghi. The principle of this method may be explained with respect to seepage flow below a sheet pile wall. Fig. 8.12 (a) is a sheet pile wall with the flow net drawn. The uplift pressures acting on a horizontal plane *ox* can be determined. The ordinates of curve *C* in Fig. 8.12 (b) represent the uplift pressure at any point on the line *ox*. It is seen that the uplift pressure is greatest close to the wall and gradually becomes less with an increase in the distance from the wall. When the upward forces of seepage on a portion of *ox* near the wall become equal to the downward forces exerted by the submerged soil, the surface of the soil rises as shown in Fig. 8.12 (a). This heave occurs simultaneously with

an expansion of the volume of the soil, which causes its permeability to increase. Additional seepage causes the sand to boil, which accelerates the flow of water and leads to complete failure. Terzaghi determined from model tests that heave occurs within a distance of about  $D/2$  (where  $D$  is the depth of penetration of the pile) from the sheet pile and the critical section  $ox$  passes through the lower edge of the sheet pile.

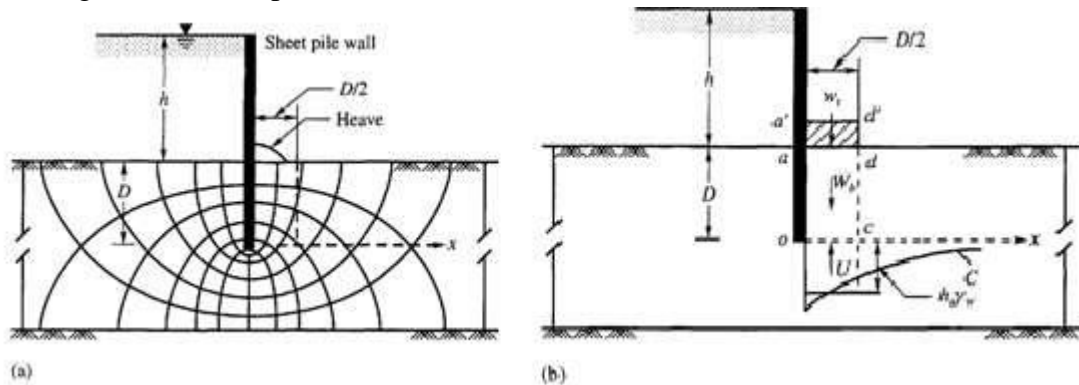


Fig. 8.12: Piping failure

### 8.11.1 Filter Requirements to Control Piping

Filter drains are required on the downstream sides of hydraulic structures and around drainage pipes. A properly graded filter prevents the erosion of soil in contact with it due to seepage forces. To prevent the movement of erodible soils into or through filters, the pore spaces between the filter particles should be small enough to hold some of the protected materials in place. Taylor (1948) shows that if three perfect spheres have diameters greater than 6.5 times the diameter of a small sphere, the small spheres can move through the larger as shown in Fig. 8.13 (a). Soils and aggregates are always composed of ranges of particle sizes, and if pore spaces in filters are small enough to hold the 85 per cent size ( $D_{85}$ ) of the protected soil in place, the finer particles will also be held in place as exhibited schematically in Fig. 8.13(b).

The requirements of a filter to keep the protected soil particles from invading the filter significantly are based on particle size. These requirements were developed from tests by Terzaghi which were later extended by the U.S. Army Corps of Engineers (1953). The resulting filter specifications relate the grading of the protective filter to that of the soil being protected by the following:

$$\frac{D_{15\text{filter}}}{D_{85\text{soil}}} \leq 4, 4 < \frac{D_{15\text{filter}}}{D_{15\text{soil}}} \leq 20 \text{ and } \frac{D_{50\text{filter}}}{D_{50\text{soil}}} \leq 25 \text{-----} \quad \text{Eq.8.57}$$

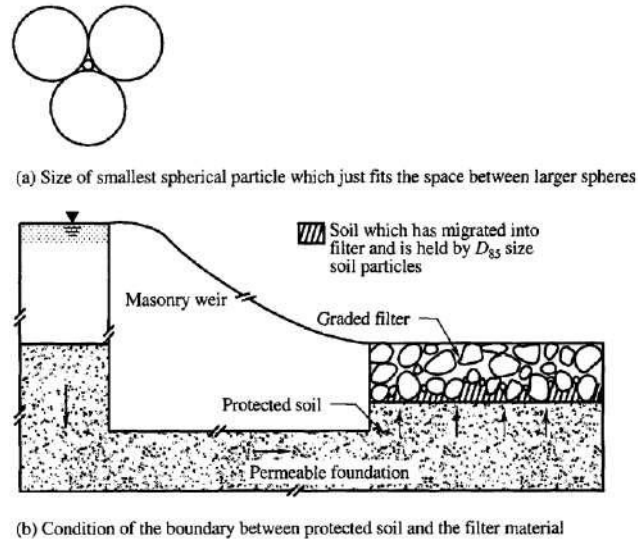


Fig.8.13: Requirements of a filter

The criteria may be explained as follows:

1. The 15 per cent size ( $D_{15}$ ) of filter material must be less than 4 times the 85 per cent size ( $D_{85}$ ) of a protected soil. The ratio of  $D_{15}$  of a filter to  $D_{85}$  of a soil is called *the piping ratio*.
2. The 15 per cent size ( $D_{15}$ ) of a filter material should be at least 4 times the 15 per cent size ( $D_{15}$ ) of a protected soil but not more than 20 times of the latter.
3. The 50 per cent size ( $D_{50}$ ) of filter material should be less than 25 times the 50 per cent size ( $D_{50}$ ) of protected soil.

Experience indicates that if the basic filter criteria mentioned above are satisfied in every part of a filter, piping cannot occur under even extremely severe conditions.

A typical grain size distribution curve of a protected soil and the limiting sizes of filter materials for constructing a graded filter is given in Figure 8.14. The size of filter materials must fall within the two curves  $C_2$  and  $C_3$  to satisfy the requirements.

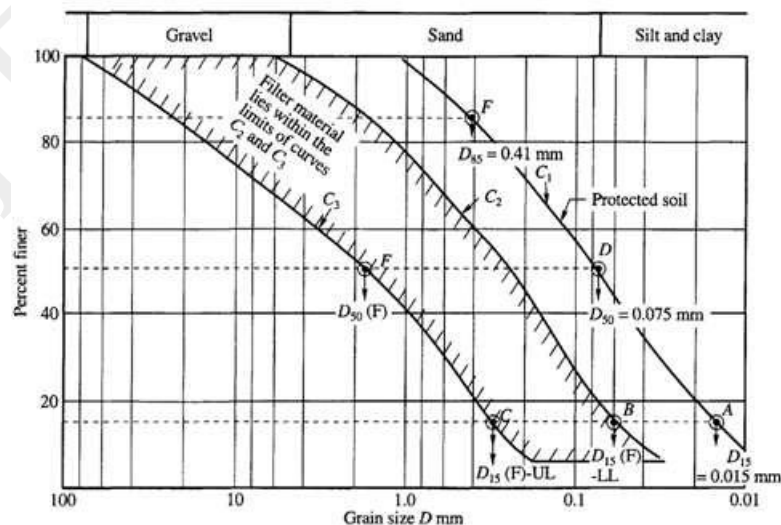


Fig.8.14: Grain size distribution curves for graded filter and protected materials

## 9.0 COMPACTION

In construction of highway embankments, earth dams and many other engineering structures, loose soils must be compacted to improve their strength by increasing their unit weight; Compaction -  
Densification of soil by removing air voids using mechanical equipment; the degree of compaction is measured in terms of its dry unit weight.

Compaction, in general, is the densification of soil by removal of air, which requires mechanical energy. The degree of compaction of a soil is measured in terms of its dry unit weight. When water is added to the soil during compaction, it acts as a softening agent on the soil particles. The soil particles slip over each other and move into a densely packed position. The dry unit weight after compaction first increases as the moisture content increases.

When the moisture content is gradually increased and the same compactive effort is used for compaction, the weight of the soil solids in a unit volume gradually increases.

Beyond a certain moisture content, any increase in the moisture content tends to reduce the dry unit weight. This phenomenon occurs because the water takes up the spaces that would have been occupied by the solid particles. The moisture content at which the maximum dry unit weight is attained is generally referred to as the *optimum moisture content*.

### 9.1 Objectives for Compaction

Increasing the bearing capacity of foundations;  
Decreasing the undesirable settlement of structures;  
Control undesirable volume changes;  
Reduction in hydraulic conductivity;  
Increasing the stability of slopes.

*In general, soil densification includes compaction and consolidation.*

**Compaction** is one kind of densification that is realized by rearrangement of soil particles without outflow of water. It is realized by application of mechanic energy. It does not involve fluid flow, but with moisture changing altering.

**Consolidation** is another kind of densification with fluid flow away. Consolidation is primarily for clayey soils. Water is squeezed out from its pores under load.

### 9.2 FACTORS AFFECTING COMPACTION

The compaction of soil depends on the following factors:

- 1 Water content
- 2 Compactive effort
- 3 Type of soil
- 4 Method of compaction

*Water content:* Water content has significant effect on compaction characteristics of soil. At low water content, soil is stiff and soil grains offer more resistance to compaction. As the water content increases, water films are formed around the soil grains and water around the soil grains act as lubricant. Due to this, soil grains come close to each other and make a dense configuration. At optimum moisture content, soil reaches the maximum unit weight as lubrication effect is the



maximum at this stage. Further addition of more water replaces the soil grains. Thus, addition of more water after optimum moisture content reduces the unit weight as unit weight of water is less than the unit weight of soil grains. Because of the due to the addition of water dry unit weight of soil increases up to a certain water content value (OMC) and beyond that addition of more water decreases the dry unit weight of soil (as shown in Fig.9.1). This can be also explained by concept of soil structure and electrical double layer theory.

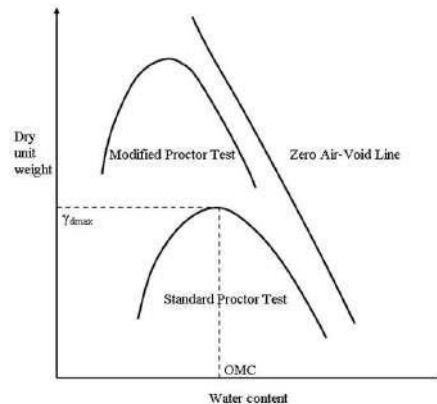
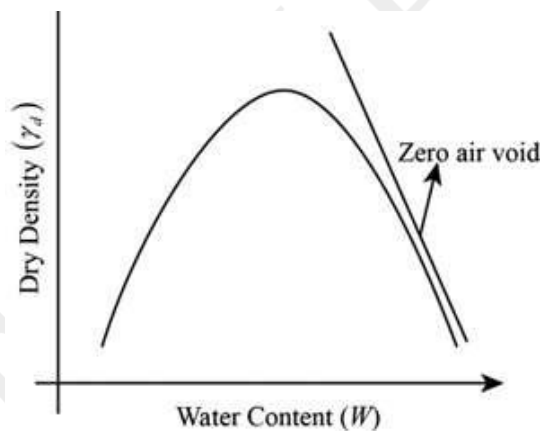


Fig.9.1: Compaction curves for clay

Zero-air voids line: It means the soil is fully saturated, with the air is not present in the pores of the soil mass. It is obtained from the compaction test. The zero-air void line drawn with the results, obtained from the compaction test as shown below.



*Compactive effort:* As the compactive effort increases maximum dry unit weight increases in a decreasing rate. However, the OMC value decreases as the compactive effort increases. Thus, if the compactive effort increases the compaction curve is shifted to the top and to the left side as shown in Figure.2. However, as the water content increases the effect of compactive effort on dry unit weight decreases. If the peaks of the compaction curves for different compactive efforts are joined the obtained line is called line of optimum (as shown in Figure 2). The line of optimum is nearly parallel to line of zero air voids. Thus, the efficiency of the compaction does not increase as the compactive effort increases.

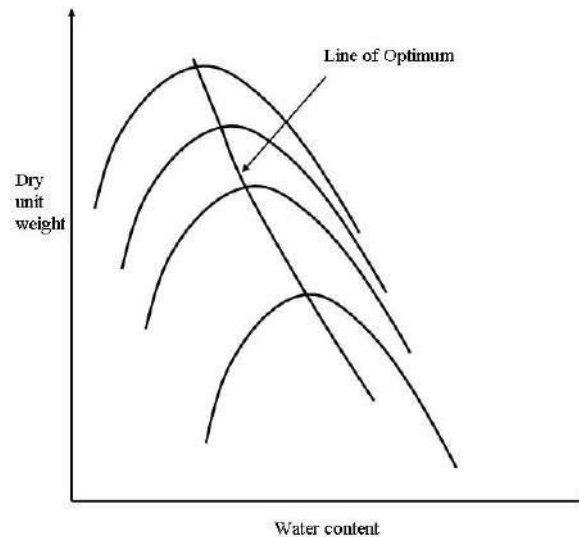


Fig.9.3: Effect of Compactive effort

From the preceding observation and Figure 9.3, we can see that

1. As the compaction effort is increased, the maximum dry unit weight of compaction is also increased.
2. As the compaction effort is increased, the optimum moisture content is decreased to some extent.

The preceding statements are true for all soils. Note, however, that the degree of compaction is not directly proportional to the compaction effort.

*Type of soil:* Soil type has significant effect on compaction. It is observed that cohesive soil has higher OMC value and poorly graded sands have lower dry unit weight as compared to the coarse-grained, well graded soil with some percentage of fines. However, excessive amount of fines reduces the maximum dry unit weight value of the soil. Maximum dry unit weight of sand is obtained either completely dry or saturated condition. Compaction curve has very little importance in case of sandy soils.

*Method of compaction:* It is observed that mode of compaction influences the shape of compaction curves. The shape of compaction curves for standard proctor test and modified proctor test are somewhat different. The shape of the compaction curve obtained from laboratory tests and obtained from field by various compactive methods are also different.

### 9.3 EFFECT OF COMPACTION ON SOIL PROPERTIES

Lambe (1958a) studied the effect of compaction on the structure of clay soils. If clay is compacted with a moisture content on the dry side of the optimum, as represented by point, it will possess a flocculent structure. This type of structure results because, at low moisture content, the diffuse double layers of ions surrounding the clay particles cannot be fully developed; hence, the inter particle repulsion is reduced. This reduced repulsion results in a more random particle orientation and a lower dry unit weight. When the moisture content of

compaction is increased, as shown by point B, the diffuse double layers around the particles expand, which increases the repulsion between the clay particles and gives a lower degree of flocculation and a higher dry unit weight. A continued increase in moisture content expands the double layers more. This expansion results in a continued increase of repulsion between the particles and thus a still greater degree of particle orientation and a more or less dispersed structure. However, the dry unit weight decreases because the added water dilutes the concentration of soil solids per unit volume. At given moisture content, higher compactive effort yields a more parallel orientation to the clay particles, which gives a more dispersed structure. The particles are closer and the soil has a higher unit weight of compaction.

Compaction induces variations in the structure of cohesive soils. Results of these structural variations include changes in hydraulic conductivity, compressibility, and strength. The hydraulic conductivity, which is a measure of how easily water flows through soil, decreases with the increase of moisture content. It reaches a minimum value at approximately the optimum moisture content. Beyond the optimum moisture content, the hydraulic conductivity increases slightly. The high value of the hydraulic conductivity on the dry side of the optimum moisture content is due to the random orientation of clay particles that results in larger pore spaces.

Some expansive clays in the field do not stay compacted, but expand upon entry of water and shrink with loss of moisture. This shrinkage and swelling of soil can cause serious distress to the foundations of structures. Laboratory observations such as this will help soils engineers to adopt a moisture content for compaction to minimize swelling and shrinkage.

When a soil is compacted, it changes its engineering properties and thereby behaves differently. Some of the engineering properties which changes on application of compactive effort is briefly described below.

#### 1. *PERMEABILITY*

The effect of compaction is to decrease the permeability. In the case of fine grained soils it has been found that for the same dry density soil compacted wet of optimum will be less permeable than that of compacted dry of optimum.

#### 2. *COMPRESSIBILITY*

In case of soil samples initially saturated and having same void ratio, it has been found that in low pressure range a wet side compacted soil is more compressible than a dry side compacted soil, and vice versa in high pressure range.

#### 3. *PORE PRESSURE*

In undrained shear test conducted on saturated samples of clay it has been found that lower pore pressures develop at low strains when the sample is compacted dry of optimum, compared to the case when the sample is compacted wet of optimum. But at high strains in both types of samples the development of pore pressure is same for same density and water content.

#### 4. *STRESS-STRAIN RELATION*

Samples compacted dry of optimum produce much steeper stress-strain curves with peaks at low strains, whereas samples compacted wet of optimum, having the same density, produce much flatter stress-strain curves with increase in stress even at high strains.

### 5. SHRINKAGE AND SWELLING

At same density a soil compacted dry of optimum shrinks appreciably less than that of compacted wet of optimum. Also the soil compacted dry of optimum exhibits greater swelling characteristics than samples of the same density compacted wet of optimum.

## 9.4 COMPACTION TESTS

Compaction test is used to establish the relationship between the moisture content and density of soils compacted in a mould of a given size with a 2.5 kg rammer dropped from a height of 30 cm. the results obtained from this test will be helpful in increasing the bearing capacity of foundations, Decreasing the undesirable settlement of structures, Control undesirable volume changes, Reduction in hydraulic conductivity, Increasing the stability of slope and so on.

### APPARATUS REQUIRED:

1. Proctor mould having a capacity of 944 cc with an internal diameter of 10.2 cm and a height of 11.6 cm. The mould shall have a detachable collar assembly and a detachable base plate.
2. Rammer: A mechanical operated metal rammer having a 5.08 cm diameter face and a weight of 2.5 kg. The rammer shall be equipped with a suitable arrangement to control the height of drop to a free fall of 30 cm.
3. Sample extruder, mixing tools such as mixing pan, spoon, towel, and spatula.
4. A balance of 15 kg capacity, Sensitive balance, Straight edge, Graduated cylinder, Moisture tins.

### PROCEDURE:

1. Take a representative oven-dried sample, approximately 5 kg in the given pan. Thoroughly mix the sample with sufficient water to dampen it with approximate water content of 4-6 %.
2. Weigh the proctor mould without base plate and collar. Fix the collar and base plate. Place the soil in the Proctor mould and compact it in 3 layers giving 25 blows per layer with the 2.5 kg rammer falling through. The blows shall be distributed uniformly over the surface of each layer.
3. Remove the collar; trim the compacted soil even with the top of mould using a straight edge and weigh to find the bulk density,  $\rho$
4. Divide the weight of the compacted specimen by 944 cc and record the result as the bulk density.
5. Remove the sample from mould and slice vertically through and obtain a small sample for water content.

6. Thoroughly break up the remainder of the material until it will pass through 4.25mm sieve as judged by the eye. Add water in sufficient amounts to increase the moisture content of the soil sample by one or two percentage points and repeat the above procedure for each increment of water added. Continue this series of determination until there is either a decrease or no change in the wet unit weight of the compacted soil.

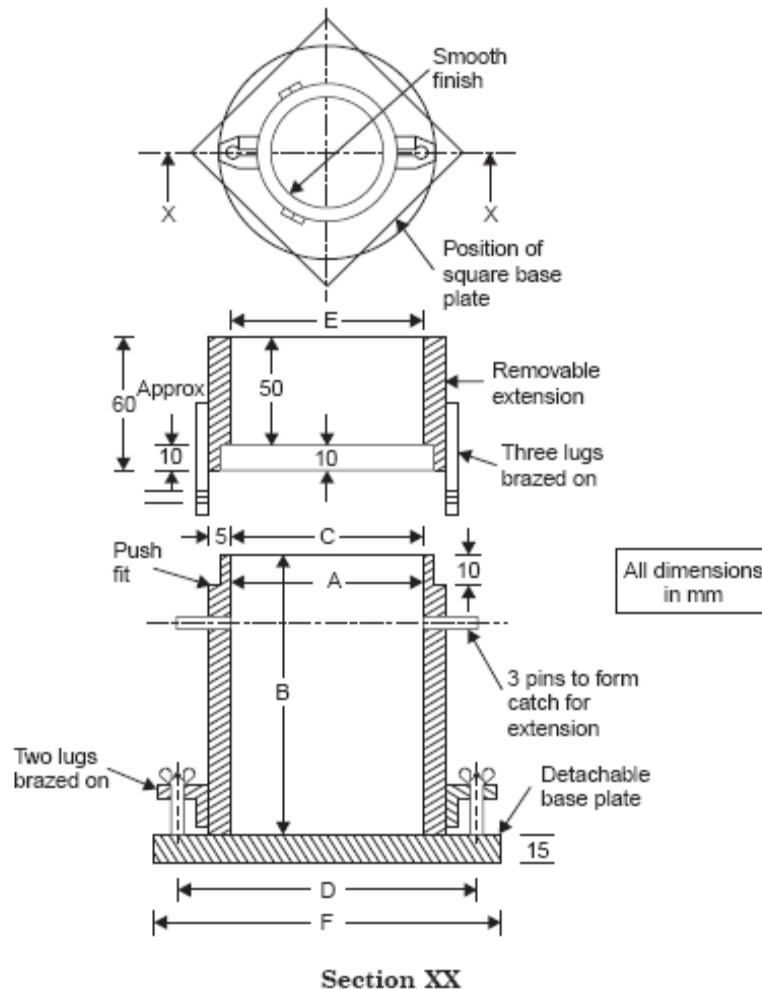


Fig.9.4: Typical mould section with dimension

Volume cm <sup>3</sup>	A dia	B	C dia	D	E dia	F
1000	100	127.3	110	150	120	180φ or 150 sq.
2250	150	127.3	160	200	170	230φ or 200 sq.

### Calculation

First, the compaction water content ( $w$ ) of the soil sample is calculated using the average of the three measurements obtained (top, middle and bottom part of the soil mass). Subsequently, the dry unit weight ( $\gamma_d$ ) is calculated as follows:

$$\gamma_d = \frac{\gamma}{1+w}$$

Eq.9.1

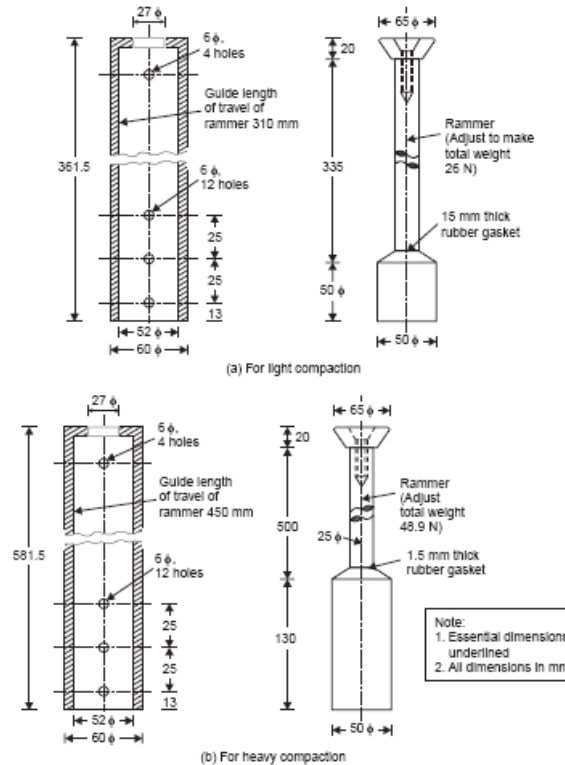


Fig.9.5: Typical section of hammer

#### 9.4.1 The Compaction Curve

The weight of each specimen is used to calculate wet unit weights and the oven-dried moistures are used to determine a dry unit weight for each point. The results are plotted on a graph as dry unit weight vs. moisture content and will show the curvilinear relationship that allows the maximum dry weight and optimum moisture for each type of soil to be established. A curve showing values at 100% saturation, or zero air voids is plotted on the same graph as detailed in the test method. This curve helps define the shape of the wet side of the maximum density curve. When performing a field density test, the dry unit weight is compared to the maximum dry weight of the Proctor tests to express a percent of compaction. When the required densities are hard to reach in the field, the moisture contents should be compared to the optimum moisture content from the lab tests. If the differences are more than 2 to 3%, it will help to adjust the field moistures accordingly by aerating/discing the soil, or by adding water.

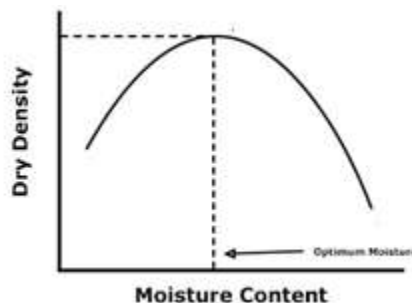


Fig.9.1: Compaction curve for Standard Proctor Test

## 9.5 MODIFIED PROCTOR TEST

### Objective

To obtain the graphical relationship of the “dry density” to “moisture content” in the form of “compaction curve“, for determining the values of Optimum Moisture Content (OMC) and Maximum Dry Density (MDD).

### Apparatus Required

Proctor Mould & Metal Rammer

Metal mould (volume =  $1000 \text{ cm}^3$  for 100 mm diameter mould and volume =  $2250 \text{ cm}^3$  for 150 mm diameter mould (as per IS: 10074-1982) & Metal rammer conforming to IS: 9189-1979. (weight = 4.9 kg)

Balance

Balance: one of capacity 10 kg and least count 1g and other of 200 g capacity and sensitivity 0.01 g

Sieves: 4.75mm, 19 mm and 37.5 mm I.S. Sieves conforming to IS: 460 (Part 1) – 1985

Oven: Thermostatically controlled to maintain temperature between  $105^0$  to  $110^0\text{C}$

Steel Straight Edge: For trimming the protruded excessive soil of the mould

Airtight Container: Taking sample for determination of Moisture Content

### Procedure

1. Take a representative portion of air-dried soil large enough to provide about 5 kg of material passing through 4.25 mm sieve.
2. Add suitable amount of water with the soil and mix it thoroughly. For sandy and gravelly soil add 3% to 5% of water. For cohesive soil the amount of water to be added should be 12% to 16% below the plastic limit.
3. Weigh the mould with base plate attached, to the nearest 1g and record the weight as W1. Attach the extension collar with the mould. Compact the moist soil into the mould in five layers of approximately equal mass, each layer being given 25 blows, with the help of 4.9 kg rammer, dropped from a height of 450mm above the soil. The blows must be distributed uniformly over the surface of each layer. The operator shall ensure that the tube of the rammer is kept clear of soil so that the rammer always falls freely.
4. After completion of the compaction operation, remove the extension collar and level carefully the top of the mould by means of straightedge. Weigh the mould with the compacted soil to the nearest 1 g and record this weight as W2.
5. Remove the compacted soil from the mould and place it on the mixing tray. Determine the water content of a representative sample of the specimen. Record the moisture content as ‘M’.
6. The remainder of the soil shall be broken up and repeat Steps (iii) to (v) above, by adding suitable increment of water to the soil. For sandy and gravelly soils the increment is

generally 1% to 2% and for cohesive soils the increment is generally 2% to 4%. The total number of determinations made shall be at least five, and the moisture contents should be such that the optimum moisture content, at which the maximum dry density occurs, is within that range.

7. For compacting soil containing coarse material up to 37.5 mm size, the 2250 cm<sup>3</sup> mould should be used. A sample weighing about 30 kg and passing the 37.5 mm IS sieve is used for the test. Soil is compacted in five layers, each layer being given 55 blows of the 4.9 kg rammer.

**NOTE 1:** It is important that the water is mixed thoroughly and adequately with the soil, since inadequate mixing gives rise to variable test results. This is particularly important with cohesive soils when adding a substantial quantity of water to the air-dried soil. With clays of high plasticity, or where hand mixing is employed, it may be difficult to distribute the water uniformly through the air-dried soil by mixing alone, and it may be necessary to store the mixed sample in a sealed container for a minimum period of about 16 hours before continuing with the test.

**NOTE 2:** It is necessary to control the total volume of soil compacted, since it has been found that if the amount of soil struck off after removing the extension is too great, the test results will be inaccurate.

**NOTE 3:** The water added for each stage of the test should be such that a range of moisture contents is obtained which includes the optimum moisture. In general, increments of 1 to 2 percent are suitable for sandy and gravelly soils and of 2 to 4 percent for cohesive soils. To increase the accuracy of the test it is often advisable to reduce the increments of water in the region of the optimum moisture content.

### Calculation

1. Bulk density,  $\gamma_m$  in g/cm<sup>3</sup> of each compacted specimen is calculated from the following equation.

$$\gamma_m = (W_2 - W_1) / V_m \text{-----} \quad \text{Eq.9.2}$$

Where,

$W_1$  = Weight in g of mould + base plate

$W_2$  = Weight in g of mould + base plate + soil

$V_m$  = Volume of mould i.e. 1000 cm<sup>3</sup>.

2. Dry density,  $\gamma_d$  in g/cm<sup>3</sup> of each compacted specimen is calculated from the following equation.

$$\gamma_d = 100 \gamma_m / (100 + w) \text{-----} \quad \text{Eq.9.3}$$



3. Where,

$\gamma_m$  = Bulk density of soil in g/cm<sup>3</sup>.

w = Moisture content of soil

### Graph

The dry densities  $\gamma_d$  obtained in a series of determinations is plotted against the corresponding moisture content 'w'. A smooth curve is then drawn through the resulting points and the position of the maximum on this curve is determined, which is called maximum dry density (M.D.D). And the corresponding moisture content is called optimum moisture content (O.M.C.).

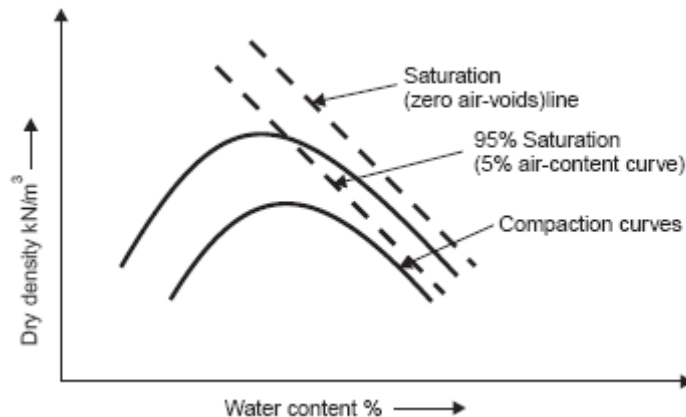


Fig. 9.2: Comparison of Compaction curve for Standard and Modified Proctor Test

### 9.6 SATURATION (ZERO-AIR-VOIDS) LINE

A line showing the relation between water content and dry density at a constant degree of saturation  $S$  may be established from the equation:

$$\gamma_d = \frac{G\gamma_w}{1 + \frac{\omega G}{S}} \quad \text{Eq.9.4}$$

Substituting  $S = 95\%$ ,  $90\%$ , and so on, one can arrive at  $\gamma_d$ -values for different values of water content in %. The lines thus obtained on a plot of  $\gamma_d$  versus  $w$  are called 95% saturation line, 90% saturation line and so on as shown in Fig.9.2.

If one substitutes  $S = 100\%$  and plots the corresponding line, one obtains the theoretical saturation line, relating dry density with water content for a soil containing no air voids. It is said to be 'theoretical' because it can never be reached in practice as it is impossible to expel the pore air completely by compaction.

We then use

$$\gamma_d = \frac{G\gamma_w}{1 + \frac{\omega G}{100}} \quad \text{Eq.9.5}$$

## 9.7 IN-SITU OR FIELD COMPACTION

For any type of construction job which requires soil to be used as a foundation material or as a construction material, compaction in-situ or in the field is necessary.

The construction of a structural fill usually consists of two distinct operations—placing and spreading in layers and then compaction. The first part assumes greater significance in major jobs such as embankments and earth dams where the soil to be used as a construction material has to be excavated from a suitable borrow area and transported to the work site. In this phase large earth moving equipment such as self-propelled scrapers, bulldozers, graders and trucks are widely employed.

The thickness of layers that can be properly compacted is known to be related to the type of soil and method or equipment of compaction. Generally speaking, granular soils can be adequately compacted in thicker layers than fine-grained soils and clays; also, for a given soil type, heavy compaction equipment is capable of compacting thicker layers than light equipment. Although the principle of compaction in the field is relatively simple, it may turn out to be a complex process if the soil in the borrow area is not at the desired optimum moisture content for compaction. The existing moisture content is to be determined and water added, if necessary. Addition of water to the soil is normally done either during excavation or transport and rarely on the construction spot; however, water must be added before excavation in the case of clayey soils. In case the soil has more moisture content than is required for proper compaction, it has to be air-dried after excavation and compacted as soon as the desired moisture content is attained.

Soil compaction or densification can be achieved by different means such as tamping action, kneading action, vibration, and impact. Compactors operating on the tamping, kneading and impact principle are effective in the case of cohesive soils, while those operating on the kneading, tamping and vibratory principle are effective in the case of cohesionless soils.

The primary types of compaction equipment are:

- 1      Rollers,
- 2      Rammers and
- 3      Vibrators.

Of these, by far the most common are rollers.

Rollers are further classified as follows:

- (a) Smooth-wheeled rollers,
- (b) Pneumatic-tyred rollers,
- (c) Sheeps -foot rollers, and
- (d) Grid rollers.

Vibrators are classified as:

- (a) Vibrating drum,

- (b) Vibrating pneumatic tyre
- (c) Vibrating plate, and
- (d) Vibro-flot

The maximum dry density sought to be achieved in-situ is specified usually as a certain percentage of the value obtainable in the laboratory compaction test. Thus control of compaction in the field requires the determination of in-situ unit weight of the compacted fill and also the moisture content.

The methods available for the determination of in-situ unit weight are:

- (a) Sand-replacement method,
- (b) Core-cutter method
- (c) Volumenometer method,
- (d) Rubber balloon method
- (e) Nuclear method,
- (f) Proctor plastic needle method.

Rapid methods of determination of moisture content such as the speedy moisture tester are adopted in this connection. Some of the above aspects are dealt with in the following subsections.

### 9.7.1 Types of In-situ Compaction Equipment

Certain types of *in-situ* compaction equipment are described below:

#### ***Rollers***

(a) *Smooth-wheeled rollers*: This type imparts static compression to the soil. There may be two or three large drums; if three drums are used, two large ones in the rear and one in the front is the common pattern. The compaction pressure is relatively low because of a large contact area. This type appears to be more suitable for compacting granular base courses and paving mixtures for highway and airfield work rather than for compacting earth fill. The relatively smooth surface obtained acts as a sort of a 'seal' at the end of a day's work and drains off rain water very well. The roller is self-propelled by a diesel engine and has a weight distribution that can be altered by the addition of ballast to the rolls. The common weight is 80 kN to 100 kN (8 to 10 t), although the range may be as much as 10 kN to 200 kN (1 to 20 t). The pressure may be of the order of 300 N (30 kg) per lineal cm of the width of rear rolls. The number of passes varies with the desired compaction; usually eight passes may be adequate to achieve the equivalent of standard Proctor compaction.

#### (b) ***Pneumatic-tyred rollers***

This type compacts primarily by kneading action. The usual form is a box or container— mounted on two axles to which pneumatic-tyred wheels are fitted; the front axle will have one wheel less than the rear and the wheels are mounted in a staggered fashion so that the entire width between the extreme wheels is covered. The weight supplied by earth ballast or other material placed in the container may range from 120 kN (12 t) to 450 kN (45 t), although an exceptionally heavy capacity of 2000 kN (200 t) may be occasionally used. Some equipment is provided with a "Wobble-wheel" effect, a design in which a slightly weaving path is tracked by

the travelling wheels; this facilitates the exertion of a steady pressure on uneven ground, which is very useful in the initial stages of a fill. The weight of the roller as well as the contact pressure is an important parameter for the performance; the latter may be varied from  $0.20$  to  $1 \text{ N/mm}^2$  ( $2$  to  $10 \text{ kg/cm}^2$ ) through the adjustment of air pressure in the tyres. Although this type has originated as a towed unit, self propelled units are also available. The number of passes required is similar to that with smooth wheeled-rollers. This type is suitable for compacting most types of soil and has particular advantages with wet cohesive materials.

(c) **Sheep foot rollers:** This type of roller consists of a hollow steel drum provided with projecting studs or feet; the compaction is achieved by a combination of tamping and kneading. The drum can be filled with water or sand to provide and control the dead weight. As rolling is done, most of the roller weight is imposed through the projecting feet. It is generally used as a towed assembly with the drums mounted either singly or in pairs; self-propelled units are also available. The feet are usually club-shaped ( $100 \times 75 \text{ mm}$ ) or tapered ( $57 \times 57 \text{ mm}$ ), the number on a  $50 \text{ kN}$  ( $5 \text{ t}$ ) roller ranging from  $64$  to  $88$ . The contact pressures of the feet may range from  $700 \text{ kN/m}^2$  ( $7 \text{ kg/cm}^2$ ) to  $4200 \text{ kN/m}^2$  ( $42 \text{ kg/cm}^2$ ) and weight per drum from  $25 \text{ kN}$  ( $2.5 \text{ t}$ ) to  $130 \text{ kN}$  ( $13 \text{ t}$ ). Initially, the projections sink into the loose soil and compact the soil near the lowest portion of the layer. In subsequent passes with the roller, the zone of compaction continues to rise until the surface is reached, when the roller is said to “Walk-out”. The length of the studs, the contact area and the weight of roller are related to the roller performance. This type of roller is found suitable for cohesive soils. It is unsuitable for granular soils as the studs tend to loosen these continuously. The tendency of void formation is more in soils compacted with sheep foot rollers.

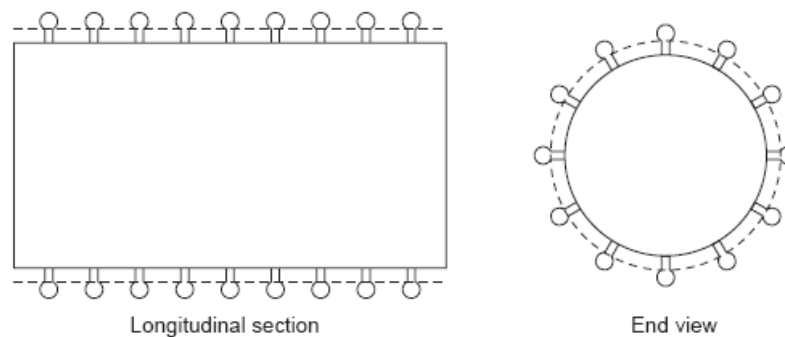


Fig. 9.3: Schematic diagram of Sheep Foot Roller

(d) **Grid rollers:** This type consists of rolls made from  $38 \text{ mm}$  steel bars at  $130 \text{ mm}$  centres, with spaces of  $90 \text{ mm}$  square. The weight of the roller ranges from  $55 \text{ kN}$  ( $5.5 \text{ t}$ ) to  $110 \text{ kN}$  ( $11 \text{ t}$ ). This is usually a towed unit which is suitable for many types of soil including wet clays and silts.

#### **Rammers**

This type includes the dropping type and pneumatic and internal commission type, which are also called ‘frog rammers’. They weigh up to about  $1.5 \text{ kN}$  ( $150 \text{ kg}$ ) and even as much as  $10 \text{ kN}$  ( $1 \text{ t}$ ) occasionally. This type may be used for cohesionless soils, especially in small restricted and confined areas such as beds of drainage trenches and back fills of bridge abutments.

### ***Vibrators***

These are vibrating units of the out-of-balance weight type or the pulsating hydraulic type. Such a type is highly effective for cohesionless soils. Behind retaining walls where the soil is confined, the backfill, much deeper in thickness, may be effectively compacted by vibration type of compactors.

A few of this type are dealt with below:

- (a) *Vibrating drum*: A separate motor drives an arrangement of eccentric weights so as to cause a high-frequency, low-amplitude, vertical oscillation to the drum. Smooth drums as well as sheep foot type of drums may be used. Layers of the order of 1 meter deep could be compacted to high densities.
- (b) *Vibrating pneumatic tyre*: A separate vibrating unit is attached to the wheel axle. The ballast box is suspended separately from the axle so that it does not vibrate. A 300 mm thick layer of granular soil will be satisfactorily compacted after a few passes.
- (c) *Vibrating plate*: This typically consists of a number of small plates, each of which is operated by a separate vibrating unit. These have a limited depth of effectiveness and hence are used in compacting granular base courses for highway and airfield pavements.
- (d) *Vibroflot*: A method suited for compacting thick deposits of loose sandy soil is called the ‘vibro flotation’ process. The improvement of density is restricted to the surface zone in the case of conventional compaction equipment. The vibroflotation method first compacts deep zone in the soil and then works its way towards the surface. A cylindrical vibrator weighing about 20 kN (2 t) and approximately 400 mm in diameter and 2 m long, called the ‘Vibroflot’, is suspended from a crane and is jetted to the depth where compaction is to start. The jetting consists of a water jet under pressure directed into the earth from the tip of the vibroflot; as the sand gets displaced, the vibroflot sinks into the soil. Depths up to 12 m can be reached. After the vibroflot is sunk to the desired depth, the vibrator is activated. The compaction of the soil occurs in the horizontal direction up to as much as 1.5 m outward from the vibroflot. Vibration continues as the vibroflot is slowly raised toward the surface. As this process goes on, additional sand is continually dropped into the space around the vibroflot to fill the void created. To densify the soil in a given site, locations at approximately 3-m spacings are chosen and treated with vibro flotation.

### **9.8 CONTROL OF COMPACTION IN THE FIELD**

Control of compaction in the field consists of checking the water content in relation to the laboratory optimum moisture content and the dry unit weight achieved *in-situ* in relation to the laboratory maximum dry unit weight from a standard compaction test. Typically, each layer is tested at several random locations after it has been compacted.

Several methods are available for the determination of *in-situ* unit weight and moisture content. The common approaches for the determination of unit weight are the core-cutter method and sand-replacement method. A faster method is what is known as the Proctor needle method, which may be used for the determination of *in-situ* unit weight as well as *in-situ* moisture content. The required density can be specified either by ‘relative compaction’ (also called

‘degree of compaction’) or by the final air-void content. Relative compaction means the ratio of the *insitu* dry unit weight achieved by compaction to the maximum dry unit weight obtained from an appropriate standard compaction test in the laboratory. Usually, the relative compaction of 90 to 100% (depending upon the maximum laboratory value), corresponding to about 5 to 10% air content, is specified and sought to be achieved. Typical values of dry unit weights achieved may be as high as  $22.5 \text{ kN/m}^3$  ( $2250 \text{ kg/m}^3$ ) for well-graded gravel and may be as low as  $14.4 \text{ kN/m}^3$  ( $1440 \text{ kg/m}^3$ ) for clays. Approximate ranges of optimum moisture content may be 6 to 10% for sands, 8 to 12% for sand-silt mixtures, 11 to 15% for silts and 13 to 21% for clays (as got from modified AASHO tests).

A variation of 5 to 10% is allowed in the field specification of dry unit weight at random locations, provided the average is about the specified value.

### 9.8.1 Proctor Needle

The Proctor needle approach given here is an efficient and fast one for the simultaneous determination of in-situ unit weight and in-situ moisture content, it is also called ‘penetration needle’. The apparatus basically consists of a needle attached to a spring-loaded plunger through a shank. An array of interchangeable needle tips is available, ranging from  $6.45$  to  $645 \text{ mm}^2$ , to facilitate the measurement of a wide range of penetration resistance values. A calibration of penetration against dry unit weight and water content is obtained by pushing the needle into specially prepared samples for which these values are known and noting the penetration. The penetration of the needle and the penetration resistance (load applied) may be shown on a graduated scale on the shank and the stem of handle respectively.

The procedure for the use of the Proctor ‘plasticity’ needle, as it is called, is obvious. The spring-loaded plunger is pressed into the compacted layer in the field with an appropriate plasticity needle. The penetration resistance is recorded for a standard depth of penetration at a standard time-rate of penetration. Against this penetration resistance, the corresponding values of water content and dry unit weight are obtained from the calibration curve.

The size of the needle to be chosen depends upon the type of soil such that the resistance to be read is neither too large nor too small. The Tennessee Valley Authority (TVA) engineers had devised a similar device, which is called the TVA ‘Penetrometer’.

### 9.8.2 Special Compaction Techniques

Several special types of compaction techniques have been developed for deep compaction of in-place soils, and these techniques are used in the field for large-scale compaction works. Among these, the popular methods are vibroflotation, dynamic compaction, and blasting. Details of these methods are provided in the following sections.

### 9.8.3 Vibro-flotation

Vibro-flotation is a technique for in situ densification of thick layers of loose granular soil deposits. It was developed in Germany in the 1930s. The first vibroflotation device was used in the United States about 10 years later. The process involves the use of a Vibroflot Unit.

This vibrating unit has an eccentric weight inside it and can develop a centrifugal force, which enables the vibrating unit to vibrate horizontally. There are openings at the bottom and top of the vibrating unit for water jets.

The vibrating unit is attached to a follow-up pipe. Figure 9.4 shows the entire assembly of equipment necessary for conducting the field compaction.

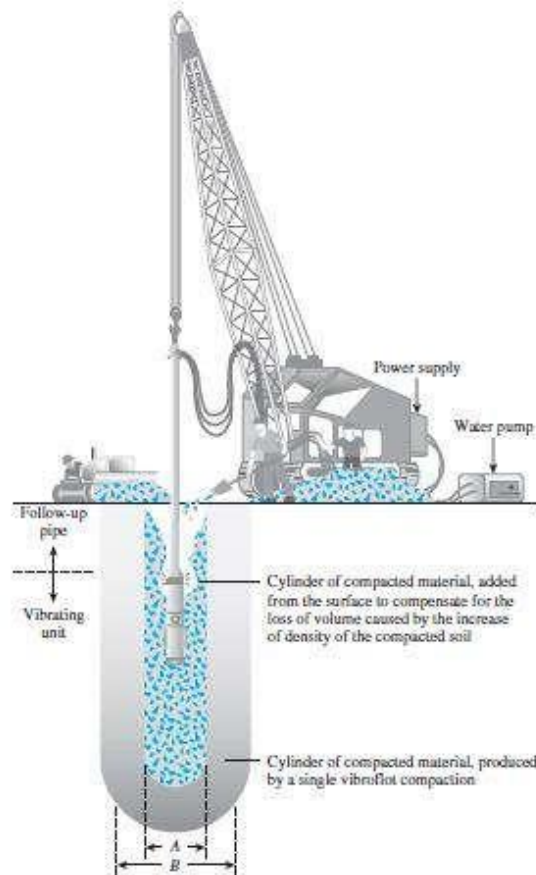


Fig.9.4: Vibro-flotation Unit

The entire vibro-flotation compaction process in the field can be divided into four stages as described in Figure 9.5.

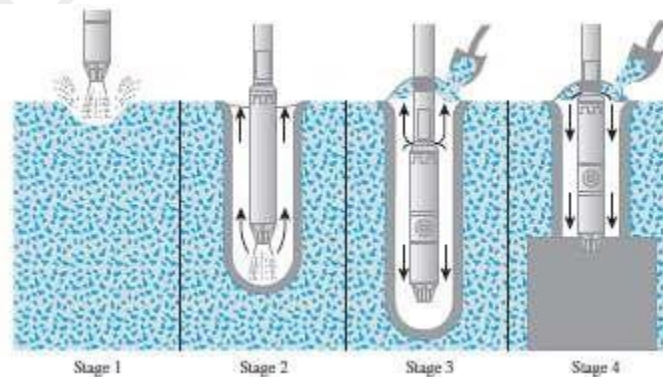


Fig.9.5: Various stages of Vibroflotation

Stage 1: The jet at the bottom of the Vibroflot is turned on and lowered into the ground.

Stage 2: The water jet creates a quick condition in the soil and it allows the vibrating unit to sink into the ground.

Stage 3: Granular material is poured from the top of the hole. The water from the lower jet is transferred to the jet at the top of the vibrating unit.

This water carries the granular material down the hole.

Stage 4: The vibrating unit is gradually raised in about 0.3 m lifts and held vibrating for about 30 seconds at each lift. This process compacts the soil to the desired unit weight.

## 9.9 DYNAMIC COMPACTION

Dynamic compaction is a technique that has gained popularity in the United States for the densification of granular soil deposits. This process consists primarily of dropping a heavy weight repeatedly on the ground at regular intervals. The weight of the hammer used varies over a range of 80 to 360 kN, and the height of the hammer drop varies between 7.5 and 30.5 m. The stress wave generated by the hammer helps in the densification.

The degree of compaction achieved at a given site depends on the following three factors:

1. Weight of hammer
2. Height of hammer drop
3. Spacing of locations at which the hammer is dropped

Figure 9.6, shows a dynamic compaction in progress. A site immediately after the completion is shown in Figure 9.6 b.

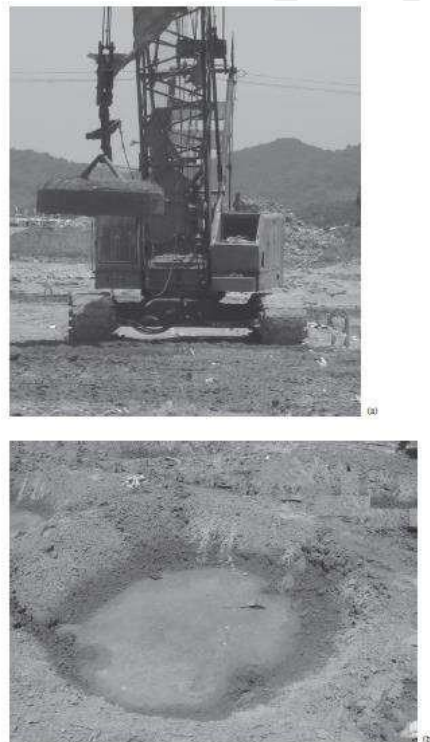


Fig.9.6 a Dynamic compaction in progress, b Site after Compaction



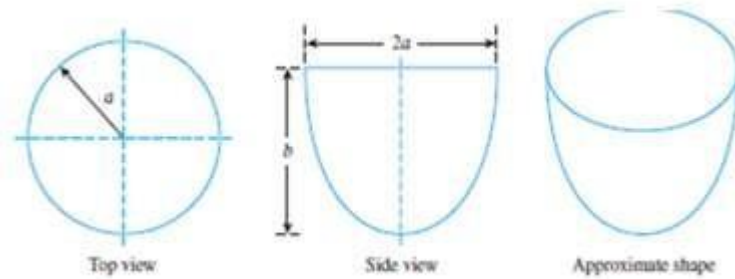


Fig.9.7: Different shapes of soil after dynamic compaction

Leonards, Cutter, and Holtz (1980) suggested that the significant depth of influence for compaction can be approximated by using the equation

$$D = \frac{1}{2} \sqrt[2]{\frac{W_H h}{H}} \quad \text{Eq.9.6}$$

where D = significant depth of densification (m)

$W_H$  = dropping weight (metric ton)

h = height of drop (m)

In English units, the preceding equation takes the form

$$D = 0.61 \sqrt{W_H h} \quad \text{Eq.9.7}$$

where the units of D and h are ft, and the unit of  $W_H$  is kip.

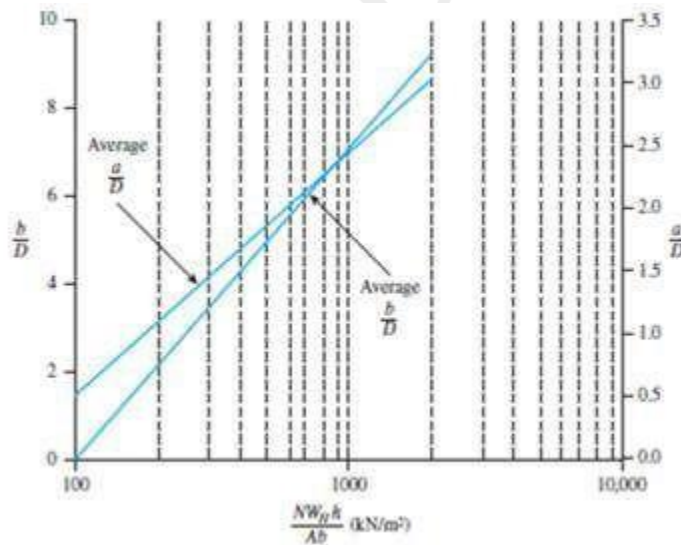


Fig.9.8: Design chart

In 1992, Poran and Rodriguez suggested a rational method for conducting dynamic compaction for granular soils in the field. According to their method, for a hammer of width D having a weight  $W_H$  and a drop h, the approximate shape of the densified area will be of the type shown in Figure 9.7, a (i.e., a semiprolate spheroid).

Note that in this figure  $b = DI$  (where  $DI$  is the significant depth of densification). Figure 9.8 gives the design chart for  $a/D$  and  $b/D$  versus  $NW_Hh/Ab$  ( $D$  is the width of the hammer if not circular in cross section;  $A$  = area of cross section of the hammer; and  $N$  is the number of required hammer drops). This method uses the following steps.

Step 1: Determine the required significant depth of densification,  $DI$  ( $b$ ).

Step 2: Determine the hammer weight ( $W_H$ ), height of drop ( $h$ ), dimensions of the cross section, and thus, the area  $A$  and the width  $D$ .

Step 3: Determine  $DI/D = (b/D)$ .

Step 4: Use Figure 6.39 and determine the magnitude of  $NW_Hh/Ab$  for the value of  $b/D$  obtained in step 3.

Step 5: Since the magnitudes of  $W_H$ ,  $h$ ,  $A$ , and  $b$  are known (or assumed) from step 2, the number of hammer drops can be estimated from the value of  $NW_Hh/Ab$  obtained from step 4.

Step 6: With known values of  $NW_Hh/Ab$ , determine  $a/D$  and thus  $a$  from Figure 9.8.

Step 7: The grid spacing,  $S_g$ , for dynamic compaction may now be assumed to be equal to or somewhat less than  $a$ . (See Fig. 9.9)

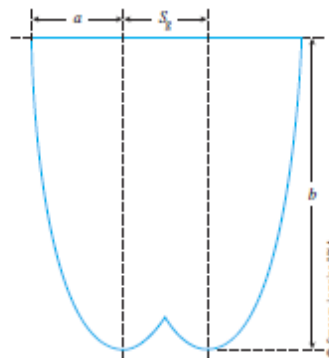


Fig. 9.9: Approximate grid spacing for dynamic compaction

## 9.10 BLASTING

Blasting is a technique that has been used successfully in many projects (Mitchell, 1970) for the densification of granular soils. The general soil grain sizes suitable for compaction by blasting are the same as those for compaction by vibroflotation.

The process involves the detonation of explosive charges, such as 60% dynamite at a certain depth below the ground surface in saturated soil. The lateral spacing of the charges varies from about 3 to 9 m. Three to five successful detonations are usually necessary to achieve the desired compaction. Compaction (up to a relative density of about 80%) up to a depth of about 18 m

over a large area can easily be achieved by using this process. Usually, the explosive charges are placed at a depth of about two-thirds of the thickness of the soil layer desired to be compacted. The sphere of influence of compaction by a 60% dynamite charge can be given as follows (Mitchell, 1970):

$$r = \sqrt{\frac{W_{ex}}{C}} \text{-----} \quad \text{Eq.9.8}$$

where,  $r$  = sphere of influence

$W_{ex}$  = weight of explosive 60% dynamite

$C = 0.0122$  when  $W_{ex}$  is in kg and  $r$  is in m

**Example 9.1:** An earth embankment is compacted at a water content of 18% to a bulk density of  $19.2 \text{ kN/m}^3$ . If the specific gravity of the sand is 2.7, find the void ratio and the degree of saturation of the compacted embankment.

Sol:

Water content,  $w = 18\%$

Bulk density,  $\gamma = 19.2 \text{ kN/m}^3$

Specific gravity,  $G = 2.7$

Dry density,  $\gamma_d = \frac{\gamma}{(1+w)} = \frac{19.2}{(1+0.18)} = 16.27 \text{ kN/m}^3$

But  $\gamma_d = \frac{G \cdot \gamma_w}{(1+e)}$ , where  $\gamma_w = 9.81 \text{ kN/m}^3$

$\therefore (1+e) = \frac{2.7 \times 9.81}{16.27} = 1.63$

Void ratio,  $e = 0.63$

Also,  $wG = S \cdot e$

$\therefore$  The degree of saturation,  $S = \frac{wG}{e} = \frac{0.18 \times 2.7}{0.63} = 0.7714$

$\therefore$  The degree of saturation = 77.14%

**Problem 1:** The soil in a borrow pit has a void ratio of 0.90. A fill-in-place volume of  $20,000 \text{ m}^3$  is to be constructed with an in-place dry density of  $18.84 \text{ kN/m}^3$ . If the owner of borrow area is to be compensated at Rs.1.50 per cubic metre of excavation, determine the cost of compensation.

## 10.0 CONSOLIDATION AND COMPRESSION OF SOILS

When a soil layer is subjected to vertical stress, volume change can take place through rearrangement of soil grains, and some amount of grain fracture may also take place. The volume of soil grains remains constant, so change in total volume is due to change in volume of water. In saturated soils, this can happen only if water is pushed out of the voids. The movement of water takes time and is controlled by the **permeability** of the soil and the locations of free draining boundary surfaces.

It is necessary to determine both the magnitude of volume change (or the settlement) and the time required for the volume change to occur. The magnitude of settlement is dependent on the magnitude of applied stress, thickness of the soil layer, and the **compressibility** of the soil.

When soil is loaded undrained, the pore pressure increases. As the excess pore pressure dissipates and water leaves the soil, settlement takes place. This process takes time, and the rate of settlement decreases over time. In coarse soils (sands and gravels), volume change occurs immediately as pore pressures are dissipated rapidly due to high permeability. In fine soils (silts and clays), slow seepage occurs due to low permeability.

**Elastic settlement** is on account of change in shape at constant volume, i.e. due to vertical compression and lateral expansion. **Primary consolidation** (or simply **consolidation**) is on account of flow of water from the voids, and is a function of the permeability and compressibility of soil. **Secondary compression** is on account of creep-like behaviour.

Primary consolidation is the major component and it can be reasonably estimated. A general theory for consolidation, incorporating three-dimensional flow is complicated and only applicable to a very limited range of problems in geotechnical engineering. For the vast majority of practical settlement problems, it is sufficient to consider that both seepage and strain take place in one direction only, as **one-dimensional consolidation** in the vertical direction.

### 10.1 Compressibility Characteristics

Soils are often subjected to uniform loading over large areas, such as from wide foundations, fills or embankments. Under such conditions, the soil which is remote from the edges of the loaded area undergoes vertical strain, but no horizontal strain. Thus, the settlement occurs only in one-dimension.

The compressibility of soils under one-dimensional compression can be described from the decrease in the volume of voids with the increase of effective stress. This relation of void ratio and effective stress can be depicted either as an **arithmetic plot** or a **semi-log plot**.

In the arithmetic plot as shown, as the soil compresses, for the same increase of effective stress  $\sigma'$ , the void ratio reduces by a smaller magnitude, from  $\Delta e_1$  to  $\Delta e_2$ . This is on account of an increasingly denser packing of the soil particles as the pore water is forced out. In fine soils, a much longer time is required for the pore water to escape, as compared to coarse soils.

It can be said that the compressibility of a soil decreases as the effective stress increases. This can be represented by the slope of the void ratio – effective stress relation, which is called the **coefficient of compressibility**,  $a_v$ .

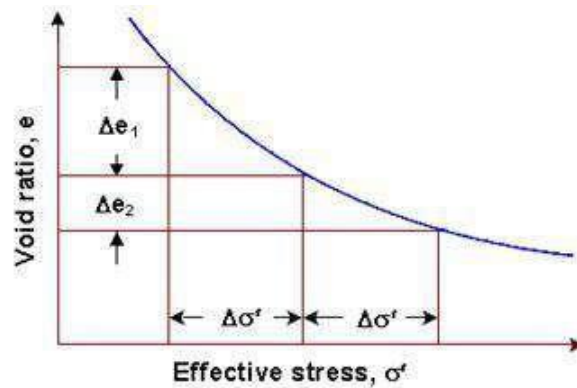


Fig.10.1: Variation of effective stress with void ratio

For a small range of effective stress,  $a_v = -\frac{\Delta e}{\Delta \sigma'}$  Eq.10.1

$$\text{or, } a_v = -\frac{de}{d\sigma'} \quad \text{Eq.10.2}$$

The -ve sign is introduced to make  $a_v$  a positive parameter.

If  $e_0$  is the initial void ratio of the consolidating layer, another useful parameter is the **coefficient of volume compressibility,  $m_v$** , which is expressed as

$$m_v = \frac{a_v}{1+e_0} \quad \text{Eq.10.3}$$

It represents the compression of the soil, per unit original thickness, due to a unit increase of pressure.

## 10.2 NORMALLY CONSOLIDATED AND OVER CONSOLIDATED

If the current effective stress,  $\sigma'$ , is equal (note that it cannot be greater than) to the pre-consolidation stress, then the deposit is said to be **normally consolidated (NC)**. If the current effective stress is less than the pre-consolidation stress, then the soil is said to be **over-consolidated (OC)**.

The figure shows the relation of void ratio and effective stress of a clay soil as a **semi-log plot**.

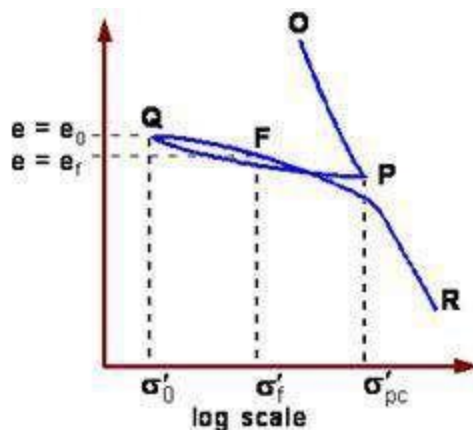


Fig.10.2: Variation of effective in logarithmic scale

**OP** corresponds to initial loading of the soil. **PQ** corresponds to unloading of the soil. **QFR** corresponds to a reloading of the soil. Upon reloading beyond **P**, the soil continues along the path that it would have followed if loaded from **O** to **R** continuously.

The **pre consolidation stress**,  $\sigma'_{pc}$  is defined to be the maximum effective stress experienced by the soil. This stress is identified in comparison with the effective stress in its present state. For soil at state **Q** or **F**, this would correspond to the effective stress at point **P**.

It may be seen that for the same increase in effective stress, the change in void ratio is much less for an over-consolidated soil (**from  $e_0$  to  $e_f$** ), than it would have been for a normally consolidated soil as in path **OP**. In unloading, the soil swells but the increase in volume is much less than the initial decrease in volume for the same stress difference.

The distance from the normal consolidation line has an important influence on soil behaviour. This is described numerically by the **over-consolidation ratio (OCR)**, which is defined as the ratio of the pre-consolidation stress to the current effective stress.

$$OCR = \frac{\sigma'_{pc}}{\sigma'} \text{-----} \quad \text{Eq.10.4}$$

Note that when the soil is normally consolidated, **OCR = 1**

Settlements will generally be much smaller for structures built on overconsolidated soils. Most soils are over-consolidated to some degree. This can be due to shrinking and swelling of the soil on drying and rewetting, changes in ground water levels, and unloading due to erosion of overlying strata.

For **NC clays**, the plot of void ratio versus log of effective stress can be approximated to a straight line, and the slope of this line is indicated by a parameter termed as **compression index,  $C_c$** .

$$C_c = \frac{\Delta e}{\log_{10} \frac{\sigma'_2}{\sigma'_1}} \text{-----} \quad \text{Eq.10.5}$$

### Estimation of Pre-Consolidation Stress

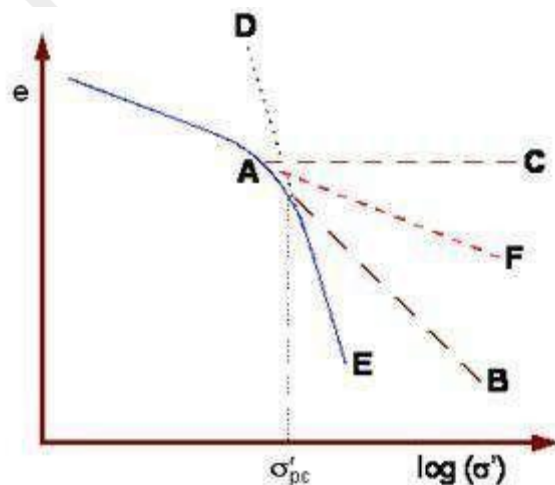


Fig.10.3: Graphical method to determine pre consolidation stress

It is possible to determine the pre-consolidation stress that the soil had experienced. The soil sample is to be loaded in the laboratory so as to obtain the void ratio - effective stress relationship. Empirical procedures are used to estimate the pre-consolidation stress, the most widely used being **Casagrande's construction** which is illustrated as

The steps in the construction are:

- Draw the graph using an appropriate scale.
- Determine the point of maximum curvature **A**.
- At **A**, draw a tangent **AB** to the curve.
- At **A**, draw a horizontal line **AC**.
- Draw the extension **ED** of the straight line portion of the curve.
- Where the line **ED** cuts the bisector **AF** of angle **CAB**, that point corresponds to the pre-consolidation stress.

The total stress increases when additional vertical load is first applied. Instantaneously, the pore water pressure increases by exactly the same amount. Subsequently there will be flow from regions of higher excess pore pressure to regions of lower excess pore pressure causing dissipation. The effective stress will change and the soil will consolidate with time. This is shown schematically.

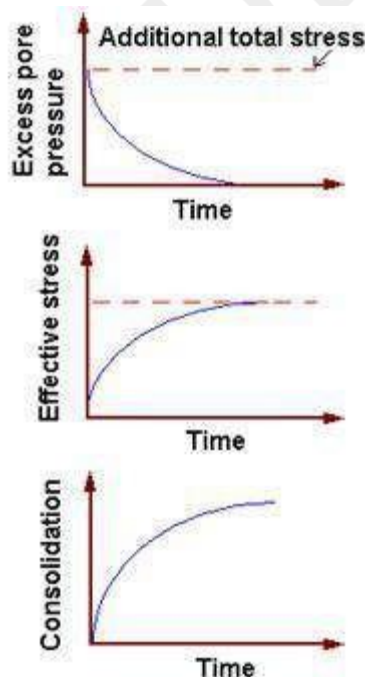


Fig.10.4: Changes of effective stress, pore water pressure and consolidation



### 10.3 SPRING ANALOGY TO EXPLAIN CONSOLIDATION THEORY

A mechanistic model for the phenomenon of consolidation was given by Taylor (1948), by which the process can be better understood. This model, with slight modifications, is presented in Fig. 10.5 and is explained below:

A spring of initial height  $H_i$  is surrounded by water in a cylinder. The spring is analogous to the soil skeleton and the water to the pore water. The cylinder is fitted with a piston of area  $A$  through which a certain load may be transmitted to the system representing a saturated soil. The piston, in turn, is fitted with a vent, and a valve by which the vent may be opened or closed.

Referring to Fig.10.5 (a), let a load  $P$  be applied on the piston. Let us assume that the valve of the vent is open and no flow is occurring. This indicates that the system is in equilibrium under the total stress  $P/A$  which is fully borne by the spring, the pressure in the water being zero.

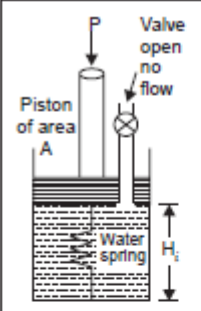
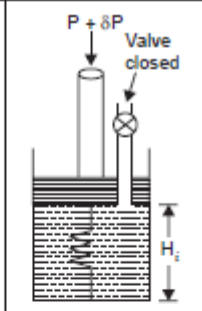
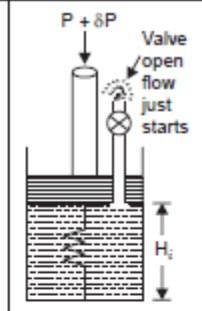
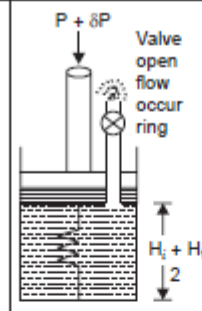
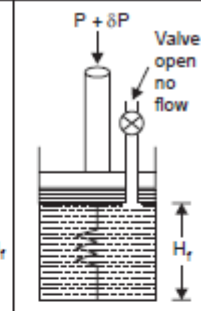
				
$\sigma = \frac{P}{A}$ $u = 0$ $\bar{\sigma} = \frac{P}{A}$	$\sigma = \frac{P}{A} + \frac{\delta P}{A}$ $u = 0 + \frac{\delta P}{A}$ $\bar{\sigma} = \frac{P}{A} + 0$	$\bar{\sigma} = \frac{P}{A} + \frac{\delta P}{A}$ $u = 0 + \frac{\delta P}{A}$ $\bar{\sigma} = \frac{P}{A} + 0$	$\sigma = \frac{P}{A} + \frac{\delta P}{A}$ $u = 0 + \frac{1}{2} \frac{\delta P}{A}$ $\bar{\sigma} = \frac{P}{A} + \frac{1}{2} \frac{\delta P}{A}$	$\sigma = \frac{P}{A} + \frac{\delta P}{A}$ $u = 0$ $\bar{\sigma} = \frac{P}{A} + \frac{\delta P}{A}$
Equilibrium under load $P$	Equilibrium under load $P + \delta P$	Beginning of transient flow; excess $u$ just starts to reduce and $\bar{\sigma}$ just starts to increase. $t = 0$ , 0% consolidation	Half-way of transient flow; 50% of excess $u$ dissipated; $\sigma$ increased by $\frac{1}{2} \cdot \frac{\delta p}{A}$ $0 < t < t_r$ 50% consolidation	End of transient flow; excess $u$ fully dissipated; $\bar{\sigma}$ increased to $\frac{P}{A} + \frac{\delta P}{A}$ $t = t_r$ equilibrium under load $(P + \delta P)$ 100% consolidation
(a)	(b)	(c)	(d)	(e)

Fig.10.5: Spring Model for Consolidation of Soil mass

Referring to Fig.10.5 (b), let us apply an increment of load  $\delta P$  to the piston, the valve being kept closed. Since no water is allowed to flow out, the piston cannot move downwards and compress the spring; therefore, the spring carries the earlier stress of  $P/A$ , while the water is forced to carry the additional stress of  $\delta P/A$  imposed on the system, the sum counteracting the total stress imposed. This additional stress  $\delta P/A$  in the water is known as the hydrostatic excess pressure.

Referring to Fig.10.5 (c), let us open the valve and start reckoning time from that instant. Water just starts to flow under the pressure gradient between it and the atmosphere seeking to return to its equilibrium or atmospheric pressure. The excess pore pressure begins to diminish, the spring starts getting compressed as the piston descends consequent to expulsion of pore water. It is just

the beginning of transient flow, simulating the phenomenon of consolidation. The openness of the valve is analogous to the permeability of soil.

Referring to Fig.10.5 (d), flow has occurred to the extent of dissipating 50% of the excess pore pressure. The pore water pressure at this instant is half the initial value, *i.e.*,  $1/2(\delta P/A)$ . This causes a corresponding increase in the stress in the spring of  $1/2(\delta P/A)$ , the total stress remaining constant at  $[(P/A) + (\delta P/A)]$ . This stage refers to that of “50% consolidation”.

Referring to Fig.10.5 (e), the final equilibrium condition is reached when the transient flow situation ceases to exist, consequent to the complete dissipation of the pore water pressure. The spring compresses to a final height  $H_f < H_i$ , carrying the total stress of  $(P + \delta P)/A$ , all by itself, since the excess pore water pressure has been reduced to zero, the pressure in it having equalled the atmospheric. The system has reached the equilibrium condition under the load  $(P + \delta P)$ . This represents “100% consolidation” under the applied load or stress increment. We may say that the “soil” has been consolidated to an effective stress of  $(P + \delta P)/A$ .

In this mechanistic model, the compressible soil skeleton is characterised by the spring and the pore water by the water in the cylinder. The more compressible the soil, the longer the time required for consolidation; the more permeable the soil, the shorter the time required. There is one important aspect in which this analogy fails to simulate consolidation of a soil. It is that the pressure conditions are the same throughout the height of the cylinder, whereas the consolidation of a soil begins near the drainage surfaces and gradually progresses inward. It may be noted that soil consolidates only when effective stress increases; that is to say, the volume change behaviour of a soil is a function of the effective stress and not the total stress.

Similar arguments may be applied to the expansion characteristics under the decrease of load.

An alternative mechanical analogy to the consolidation process is shown in Fig. 10.6. A cylinder is fitted with a number of pistons connected by springs to one another. Each of the compartments thus formed is connected to the atmosphere with the aid of standpipes. The cylinder is full of water and is considered to be airtight. The pistons are provided with perforations through which water can move from one compartment to another. The topmost piston is fitted with valves which may open or close to the atmosphere. It is assumed that any pressure applied to the top piston gets transmitted undiminished to the water and springs.

Initially, the cylinder is full of water and weights of the pistons are balanced by the springs; the water is at atmospheric pressure and the valves may be open. The water level stands at the elevation **PP** in the standpipes as shown. The valves are now closed, the water level continuing to remain at **PP**. An increment of pressure  $\Delta\sigma$  is applied on the top piston. It will be observed that the water level rises instantaneously in all the stand pipes to an elevation

**QQ**, above **PP** by a height  $h = \Delta\sigma/\gamma_w$ . Let all the valves be opened simultaneously with the application of the pressure increment, the time being reckoned from that instant. The height of the springs remains unchanged at that instant and the applied increment of pressure is fully taken up by water as the hydrostatic excess pressure over and above the atmospheric. An equal rise of water in all the standpipes indicates that the hydrostatic excess pressure is the same in all compartments immediately after application of pressure. As time elapses, the water level in the

pipes starts falling, the pistons move downwards gradually and water comes out through the open valves.

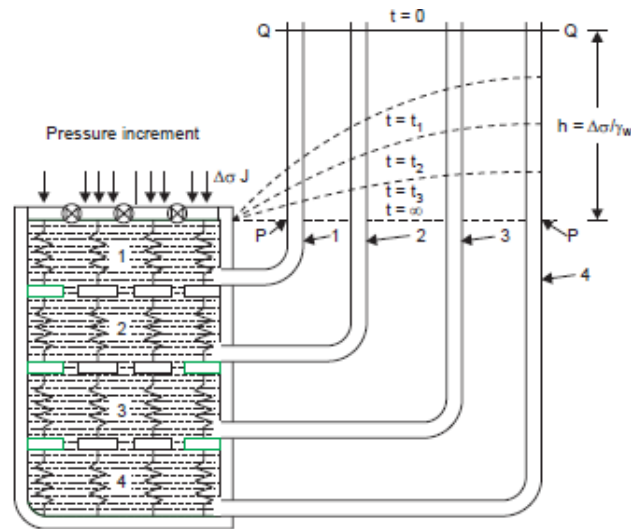


Fig.10.6: Mechanical Analogy to Consolidation process

At any time  $t = t_1$ , the water pressure in the first compartment is least and that in the last or the bottommost is highest, as indicated by the water levels in the standpipes. The variation of hydrostatic excess pressure at various points in the depth of the cylinder, as shown by the dotted lines, varies with time. Ultimately, the hydrostatic excess pressure reduces to zero in all compartments, the water levels in the standpipes reaching elevation **PP**; this theoretically speaking, is supposed to happen after the lapse of infinite time. As the hydrostatic excess pressure decreases in each compartment, the springs in each compartment experience a corresponding pressure and get compressed. For example, at time  $t = t_1$ , the hydrostatic excess pressure in the first compartment is given by the head **PJ**; the pressure taken by the springs is indicated by the head **JQ**, the sum of the two at all times being equivalent to the applied pressure increment; that is to say, it is analogous to the effective stress principle:  $\sigma = \sigma' + u$ , the pressure transferred to the springs being analogous to intergranular or effective stress in a saturated soil, and the hydrostatic excess pressure to the neutral pressure or excess pore water pressure.

Since water is permitted to escape only at one end, it is similar to the case of a single drainage face for a consolidating clay sample. The distribution of hydrostatic excess pressure will be symmetrical about mid-depth for the situation of a double drainage face, the maximum occurring at mid-depth and the minimum or zero values occurring at the drainage faces.

#### 10.4 TERZAGHI'S THEORY OF ONE-DIMENSIONAL CONSOLIDATION

Terzaghi (1925) advanced his theory of one-dimensional consolidation based upon the following assumptions, the mathematical implications being given in parentheses:

1. The soil is homogeneous ( $kz$  is independent of  $z$ ).
2. The soil is completely saturated ( $S = 100\%$ ).

3. The soil grains and water are virtually incompressible ( $\gamma_w$  is constant and volume change of soil is only due to change in void ratio).
4. The behaviour of infinitesimal masses in regard to expulsion of pore water and consequent consolidation is no different from that of larger representative masses (Principles of calculus may be applied).
5. The compression is one-dimensional ( $u$  varies with  $z$  only).
6. The flow of water in the soil voids is one-dimensional, Darcy's law being valid.

$$\frac{\partial v_x}{\partial x} = \frac{\partial v_y}{\partial y} = 0 \text{ and } v_z = k \frac{\partial h}{\partial z} \text{-----} \quad \text{Eq.10.6}$$

Also, flow occurs on account of hydrostatic excess pressure ( $h = u/\gamma_w$ ).

7. Certain soil properties such as permeability and modulus of volume change are constant; these actually vary somewhat with pressure. ( $k$  and  $m_v$  are independent of pressure).
8. The pressure versus void ratio relationship is taken to be the idealised one,  $a_v$  is constant).
9. Hydrodynamic lag alone is considered and plastic lag is ignored, although it is known to exist. (The effect of  $k$  alone is considered on the rate of expulsion of pore water).

The first three assumptions represent conditions that do not vary significantly from actual conditions.

The fourth assumption is purely of academic interest and is stated because the differential equations used in the derivation treat only infinitesimal distances. It has no significance for the laboratory soil sample or for the field soil deposit.

The fifth assumption is certainly valid for deeper strata in the field owing to lateral confinement and is also reasonably valid for an oedometer sample.

The sixth assumption regarding flow of pore water being one-dimensional may be taken to be valid for the laboratory sample, while its applicability to a field situation should be checked. However, the validity of Darcy's law for flow of pore water is unquestionable.

The seventh assumption may introduce certain errors in view of the fact that certain soil properties which enter into the theory vary somewhat with pressure but the errors are considered to be of minor importance.

The eighth and ninth assumptions lead to the limited validity of the theory. The only justification for the use of the eighth assumption is that, otherwise, the analysis becomes unduly complex. The ninth assumption is necessitated because it is not possible to take the plastic lag into account in this theory. These two assumptions also may be considered to introduce some errors.

Now let us see the derivation of Terzaghi's theory with respect to the laboratory oedometer sample with double drainage as shown in Fig. 10.7.

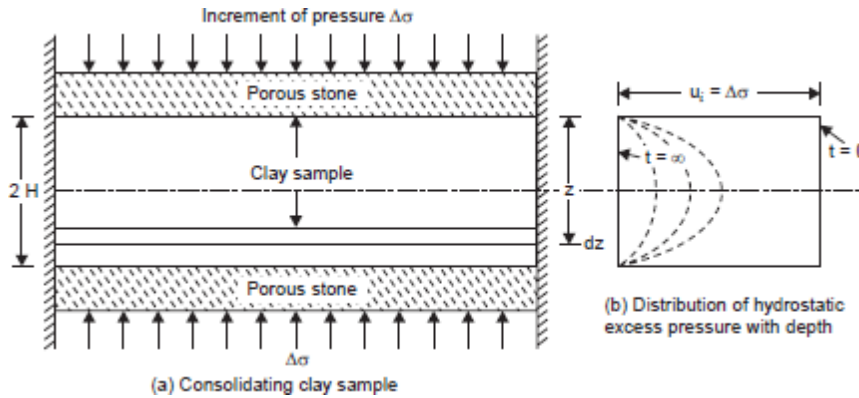


Fig.10.7: Consolidation of clay soil sample with double drainage

Let us consider a layer of unit area of cross-section and of elementary thickness  $dz$  at depth  $z$  from the pervious boundary. Let the increment of pressure applied be  $\Delta\sigma$ . immediately on application of the pressure increment, pore water starts to flow towards the drainage faces. Let  $\partial h$  be the head lost between the two faces of this elementary layer, corresponding to a decrease of hydrostatic excess pressure  $\partial u$ .

Equation 2, for flow of water through soil, holds here also,

$$k \frac{\partial^2 h}{\partial x^2} + k \frac{\partial^2 h}{\partial z^2} = \frac{1}{1+e} \left[ e \frac{\partial s}{\partial t} + S \frac{\partial e}{\partial t} \right] \quad \text{Eq.10.7}$$

For one-dimensional flow situation, this reduces to:

$$k \frac{\partial^2 h}{\partial z^2} = \frac{1}{1+e} \left[ e \frac{\partial s}{\partial t} + S \frac{\partial e}{\partial t} \right] \quad \text{Eq.10.8}$$

During the process of consolidation, the degree of saturation is taken to remain constant at 100%, while void ratio changes causing reduction in volume and dissipation of excess hydrostatic pressure through expulsion of pore water; that is,

$S=1$  or unity and  $\frac{\partial s}{\partial t} = 0$

$$\text{Hence } k \frac{\partial^2 h}{\partial z^2} = - \frac{1}{1+e} \left[ \frac{\partial e}{\partial t} \right] = \frac{\partial}{\partial t} \left[ \frac{e}{1+e} \right] \quad \text{Eq.10.9}$$

Negative sign denoting decrease of  $e$  for increase of  $h$ .

Since volume decrease can be due to a decrease in the void ratio only as the pore water and soil grains are virtually incompressible,  $\frac{\partial}{\partial t} \left[ \frac{e}{1+e} \right]$  represents time-rate of volume change per unit

volume

The flow is only due to the hydrostatic excess pressure,  $h = \frac{u}{\gamma_w}$

$$\text{So } \frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = - \frac{\partial V}{\partial t} \quad \text{Eq.10.10}$$

Here  $k$  is the permeability of soil in the direction of flow, and  $\partial V$  represents the change in volume

per unit volume. The change in hydrostatic excess pressure,  $\partial u$ , changes the intergranular or effective stress by the same magnitude, the total stress remaining constant.

The change in volume per unit volume,  $\partial V$ , may be written, as per the definition of the modulus of volume change,  $m_v$

$$\partial V = m_v \partial \sigma = -m_v \partial u \text{-----} \quad \text{Eq.10.11}$$

Negative sign is used, since increase in stress reduces pore water pressure.

Differentiating both sides with respect to time,

$$\frac{\partial V}{\partial t} = -m_v \frac{\partial u}{\partial t} \quad \text{Eq.10.12}$$

From Eq. 10.11 and 10.12, we get

$$\frac{\partial u}{\partial t} = \frac{k}{\gamma_w m_v} \frac{\partial^2 u}{\partial z^2} \quad \text{Eq.10.13}$$

This is written as:

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \quad \text{Eq.10.14}$$

Where  $c_v = \frac{k}{\gamma_w m_v}$

$c_v$  is known as the “Coefficient of consolidation”.  $u$  represents the hydrostatic excess pressure at a depth  $z$  from the drainage face at time  $t$  from the start of the process of consolidation.

The coefficient of consolidation may also be written in terms of the coefficient of compressibility

$$c_v = \frac{k}{\gamma_w m_v} = \frac{k(1+e_0)}{a_v \gamma_w} \quad \text{Eq.10.15}$$

Equation 10.14 is the basic differential equation of consolidation according to Terzaghi's theory of one-dimensional consolidation. The coefficient of consolidation combines the effect of permeability and compressibility characteristics on volume change during consolidation. Its units can be shown to be  $\text{mm}^2/\text{s}$  or  $\text{L}^2 \text{T}^{-1}$ .

The initial hydrostatic excess pressure,  $u_i$ , is equal to the increment of pressure  $\Delta\sigma$ , and is the same throughout the depth of the sample, immediately on application of the pressure, and is shown by the heavy line in Fig. 10.7 (b). The horizontal portion of the heavy line indicates the fact that, at the drainage face, the hydrostatic excess pressure instantly reduces to zero, theoretically speaking. Further, the hydrostatic excess pressure would get fully dissipated throughout the depth of the sample only after the lapse of infinite time\*, as indicated by the heavy vertical line on the left of the figure. At any other instant of time, the hydrostatic excess pressure will be maximum at the farthest point in the depth from the drainage faces, that is, at the middle and it is zero at the top and bottom. The distribution of the hydrostatic excess pressure with depth is sinusoidal at other instants of time, as shown by dotted lines. These curves are called “Isochrones”.

#### 10.4.1 Alternative Method:

With reference to Fig.10.7, the hydraulic gradient  $i_z$  at depth  $z$

$$\frac{\partial h}{\partial z} = \frac{1}{\gamma_w} \frac{\partial u}{\partial z} \quad \text{Eq.10.16}$$

$$\text{The hydraulic gradient } i \text{ at depth } z + \partial z = \frac{1}{\gamma_w} \left( \frac{\partial u}{\partial z} + \frac{\partial^2 u}{\partial z^2} \partial z \right) \quad \text{Eq.10.17}$$

Rate of inflow per unit area = Velocity at depth  $z = k.i_1$ , by Darcy's law.

Rate of outflow per unit area = velocity at  $(z + dz) = k.i_2$

Water lost per unit time =  $k(i_1 - i_2) = \frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} dz$ ----- Eq.10.18

$$\frac{\partial^2 u}{\partial z^2}$$

This should be the same as the time-rate of volume decrease.

Volumetric strain =  $m_v \Delta \sigma = -m_v \partial(\sigma - u)$ , by definition of the modulus of volume change,  $m_v$



(The negative sign denotes decrease in volume with increase in pressure).

∴ Change of volume =  $-m_v \partial(\sigma - u).dz$ , since the elementary layer of thickness  $dz$  and unit cross-sectional area is considered.

$$\text{Time-rate of change of volume} = -m_v \frac{\partial}{\partial t} (\sigma - u) dz \text{-----} \quad \text{Eq.10.19}$$

But  $\frac{\partial \sigma}{\partial t} = 0$ , since  $\sigma$  is constant.

$$\therefore \text{Time-rate of change of volume} = +m_v \frac{\partial u}{\partial t} dz \text{-----} \quad \text{Eq.10.20}$$

Equating this to water lost per unit time

$$\frac{k}{\gamma_w m_v} \frac{\partial^2 u}{\partial z^2} dz = -m_v \frac{\partial u}{\partial t} dz \text{-----} \quad \text{Eq.10.21}$$

$$\frac{\partial^2 u}{\partial z^2} = -c \frac{\partial u}{\partial t} \text{-----} \quad \text{Eq.10.22}$$

$$\frac{\partial^2 u}{\partial z^2} = -c \frac{\partial u}{\partial t}$$

$$\text{Where } c = \frac{k}{\gamma_w m_v} \text{-----} \quad \text{Eq.10.23}$$

## 10.5 SOLUTION OF TERZAGHI'S EQUATION FOR ONE-DIMENSIONAL CONSOLIDATION

Terzaghi solved the differential equation for a set of boundary conditions which have utility in solving numerous engineering problems and presented the results in graphical form using dimensional parameters.

The following are the boundary conditions:

1. There is drainage at the top of the sample: At  $z = 0$ ,  $u = 0$ , for all  $t$ .
2. There is drainage at the bottom of the sample: At  $z = 2H$ ,  $u = 0$ , for all  $t$ .
3. The initial hydrostatic excess pressure  $u_i$  is equal to the pressure increment,  $\Delta\sigma$ ,  $u = u_i = \Delta\sigma$ , at  $t = 0$ .

Terzaghi chose to consider this situation where  $u = u_i$  initially *throughout the depth*, although solutions are possible when  $u_i$  varies with depth in any specified manner. The thickness of the sample is designated by  $2H$ , the distance  $H$  thus being the length of the longest drainage path, *i.e.*, maximum distance water has to travel to reach a drainage face because of the existence of two drainage faces. (In the case of only one drainage face, this will be equal to the total thickness of the clay layer).

The general solution for the above set of boundary conditions has been obtained on the basis of separation of variables and Fourier series expansion and is as follows:

$$u = f(z, t) = \sum_{n=1}^{\infty} \left[ \int_0^{2H} u_i \sin \frac{n\pi z}{2H} dz \right] \left( \sin \frac{n\pi z}{2H} \right) e^{-n^2 \pi^2 c_v t / 4H^2} \text{--} \quad \text{Eq.10.24}$$

This solution enables the hydrostatic excess  $u$  to be computed for a soil mass under any initial system of stress  $u_i$ , at any depth  $z$ , and at any time  $t$ .

In particular, if  $u_i$  is considered constant with respect to depth, this equation reduces to

$$u = \sum_{n=1}^{\infty} \left[ \frac{2u_i}{n\pi} (1 - \cos n\pi) \left( \sin \frac{n\pi z}{2H} \right) e^{-n^2 \pi^2 c_v t / 4H^2} \right] \text{-----} \quad \text{Eq.10.25}$$

$$n=1 \quad n\pi \quad 2H$$

When  $n$  is even,  $(1 - \cos n\pi)$  vanishes; when  $n$  is odd, this factor becomes 2. Therefore it is convenient to replace  $n$  by  $(2m + 1)$ ,  $m$  being an integer. Thus, we have

$$u = \sum_{m=1}^{\infty} \left[ \frac{4u_i}{(2m+1)\pi} \left( \sin \frac{(2m+1)\pi x}{2H} \right) e^{-(2m+1)^2 \pi^2 c_v t / 4H^2} \right] \text{-----} \quad \text{Eq.10.26}$$

Three-dimensionless parameters are introduced for convenience in presenting the results in a form usable in practice. The first is  $z/H$ , relating to the location of the point at which consolidation is considered,  $H$  being the maximum length of the drainage path. The second is the consolidation ratio,  $U_z$ , to indicate the extent of dissipation of the hydrostatic excess pressure in relation to the initial value:

$$U_z = \frac{u_i - u}{u_i} = (1 - \frac{u}{u_i}) \text{-----} \quad \text{Eq.10.27}$$

The subscript  $z$  is significant, since the extent of dissipation of excess pore water pressure is different for different locations, except at the beginning and the end of the consolidation process. The third dimensionless parameter, relating to time, and called 'Time-factor',  $T$ , is defined as follows:

$$T = \frac{c_v t}{H^2} \text{-----} \quad \text{Eq.10.28}$$

Where  $c_v$  is the coefficient of consolidation,

$H$  is the length drainage path, and  $t$  is the elapsed time from the start of consolidation process.

In the context of consolidation process at a particular site,  $c_v$  and  $H$  are constants, and the time factor is directly proportional to time. Introducing the time factor we have

$$u = \sum \frac{2u_i}{M} \left( \sin \frac{M_z}{H} \right) e^{-M^2 T} \text{-----} \quad \text{Eq.10.29}$$

Introducing the consolidation ratio,  $U_z$ , we have:

$$U_z = 1 - \frac{u}{u_i} = 1 - \sum_{m=0}^{\infty} \frac{2}{M} \left( \sin \frac{M_z}{H} \right) e^{-M^2 T} \text{-----} \quad \text{Eq.10.30}$$

The following approximate expressions have been found to yield values for  $T$  with good degree of precision:

$$\text{When } U < 60\%, T = (\pi/4) U^2 \text{-----} \quad \text{Eq.10.31}$$

$$\text{When } U > 60\%, T = -0.9332 \log_{10} (1 - U) - 0.0851 \text{-----} \quad \text{Eq.10.32}$$

## 10.6 LABORATORY CONSOLIDATION TESTS

The one-dimensional consolidation testing procedure was first suggested by Terzaghi. This test is performed in a consolidometer (sometimes referred to as an *oedometer*). The schematic diagram of a consolidometer is shown in Figure 10.8(a). Figure 10.8(b) shows a photograph of a consolidometer. The soil specimen is placed inside a metal ring with two porous stones, one at the top of the specimen and another at the bottom. The specimens are usually 64 mm in diameter and 25 mm thick. The load on the specimen is applied through a lever arm, and compression is measured by a micrometer dial gauge. The specimen is kept under water during the test. Each load usually is kept for 24 hours. For consolidation testing, it is generally desirable that the applied pressure at **any loading stage be double** than that at the preceding stage. The test may, therefore, be continued using a loading sequence which would successively apply stress of 0.1, 0.2, 0.4, 0.8, 1.6, 3.2, etc, kgf/cm<sup>2</sup> on the soil specimen. For each loading increment, after application of load, readings of the dial gauge shall be taken using a time sequence such as 0, 0.25, 1, 2.25, 4.6, 25.9, 12.25, 16, 20.25, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225 min etc,

up to 24. These time sequences facilitate plotting of thickness or change of thickness of specimen

against square root of time or against logarithm of time. The loading increment shall be left at least until the slope of the characteristic linear secondary compression portion of thickness versus log time plot is apparent or until the end of pre-consolidation is indicated on a square root of time plot. A period of 24 h will usually be sufficient, but longer times may be required. In every case, the same load increment duration shall be used for all load increments during a consolidation test.

On completion of the final loading stage, the specimen shall be unloaded by pressure decrements which decrease the load to **one-fourth** of the last load. Dial gauge readings may be taken as necessary during each stage of unloading. If desired, the time intervals used during the consolidation increments may be adopted; usually it is possible to proceed much more rapidly. After the completion of test, dry weight of the soil sample is taken.

Figure 1(c) shows a consolidation test in progress. The general shape of the plot of deformation of the specimen against time for a given load increment is shown in Figure 10.9. From the plot, we can observe three distinct stages, which may be described as follows:

*Stage I:* Initial compression, which is caused mostly by preloading

*Stage II:* Primary consolidation, during which excess pore water pressure gradually is transferred into effective stress because of the expulsion of pore water

*Stage III:* Secondary consolidation, which occurs after complete dissipation of the excess pore water pressure, when some deformation of the specimen takes place because of the plastic readjustment of soil fabric

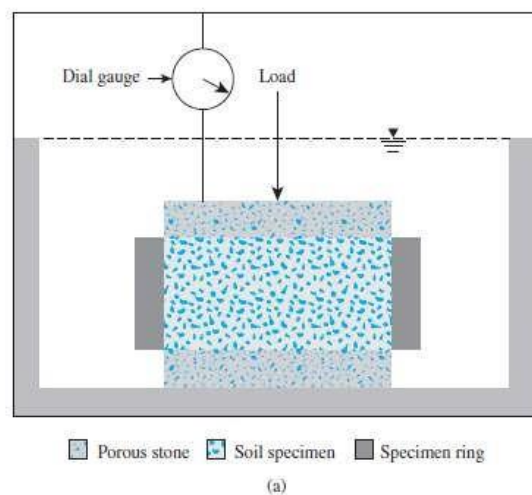
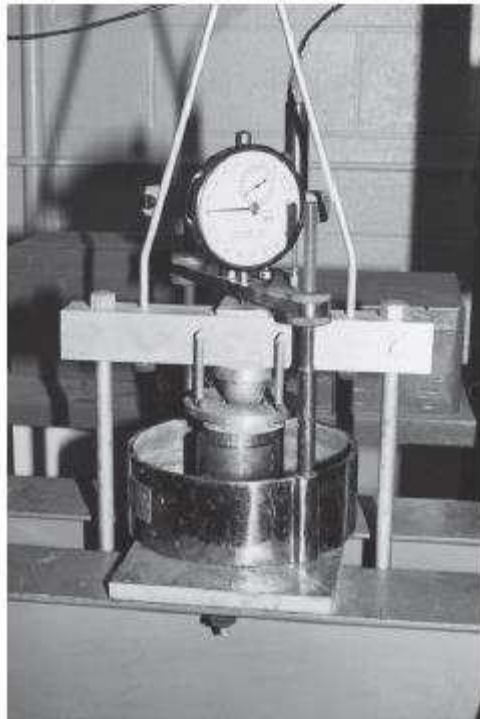
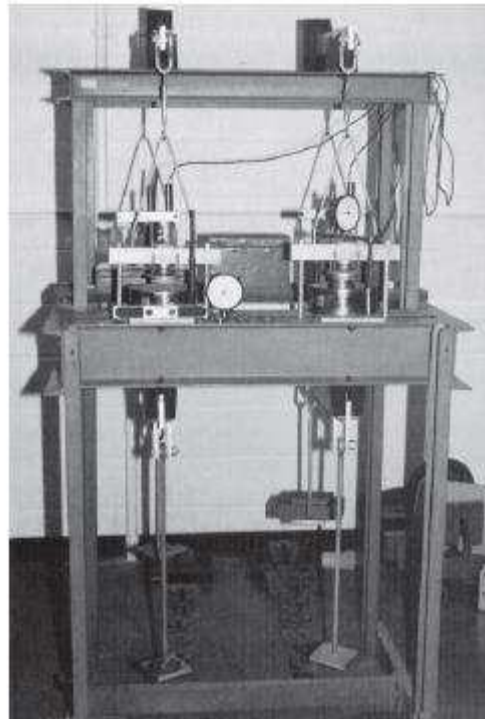


Fig. 10.8 (a): Schematic diagram of a consolidometer



(b)



(c)

Fig. 10.8 (b): Photograph of a consolidometer Fig. 10.8 (c): Consolidation test in progress

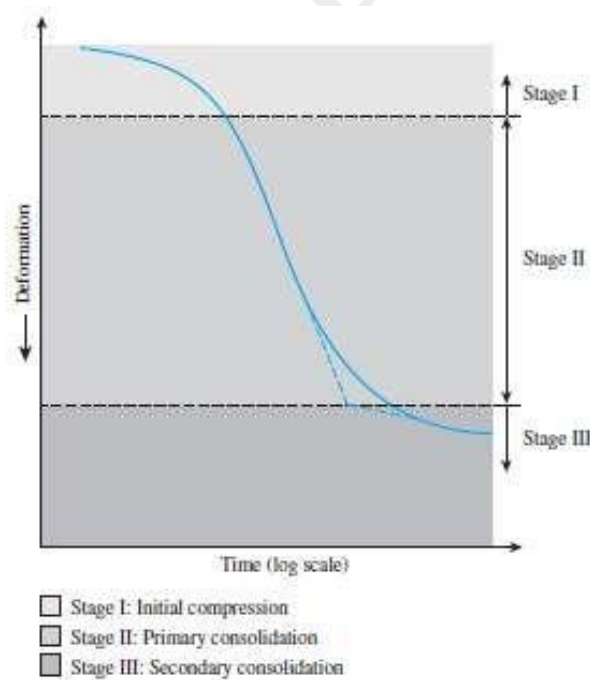


Fig. 10.9: Shape of the plot of deformation of the specimen

### 10.6.1 Void Ratio–Pressure Plots

After the time–deformation plots for various loadings are obtained in the laboratory, it is necessary to study the change in the void ratio of the specimen with pressure. Following is a step-by-step procedure for doing so:

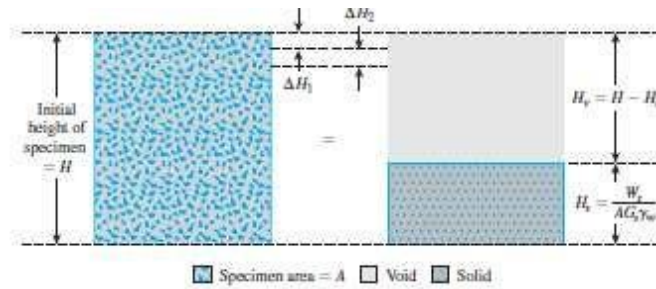


Fig. 10.10: Change of height of specimen in one-dimensional consolidation test

*Step 1:* Calculate the height of solids,  $H_s$ , in the soil specimen (Figure 10.10) using the equation

$$H_s = \frac{W_s}{A G_s \gamma_w} = \frac{M_s}{A G_s \rho_w} \quad \text{Eq.10.33}$$

where  $W_s$  = dry weight of the specimen

$M_s$  = dry mass of the specimen

$A$  = area of the specimen

$G_s$  = specific gravity of soil solids

$\gamma_w$  = unit weight of water

$\rho_w$  = density of water

*Step 2:* Calculate the initial height of voids as

$$H_v = H - H_s \quad \text{Eq.10.34}$$

Where,  $H_v$  = initial height of the specimen.

*Step 3:* Calculate the initial void ratio, of the specimen, using the equation

$$e = \frac{V_v}{V_s} = \frac{H_v A}{H_s A} = \frac{H_v}{H_s} \quad \text{Eq.10.35}$$

*Step 4:* For the first incremental loading,  $s_1$  (total load/unit area of specimen), which causes a deformation  $\Delta H_1$ , calculate the change in the void ratio as

$$\Delta e_1 = \frac{\Delta H_1}{H_s} \quad \text{Eq.10.36}$$

$\Delta H_1$  is obtained from the initial and the final dial readings for the loading).

It is important to note that, at the end of consolidation, total stress  $\sigma_1$  is equal to effective stress

*Step 5:* Calculate the new void ratio after consolidation caused by the pressure increment as

$$e_1 = e_0 - \Delta e_1 \quad \text{Eq.10.37}$$

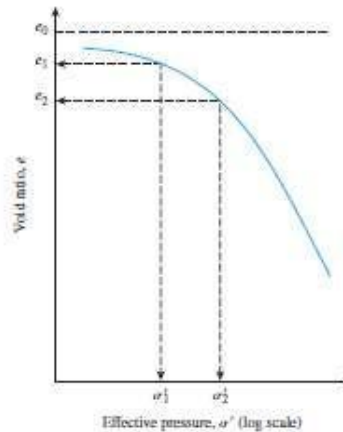
For the next loading,  $\sigma_2$  (note:  $\sigma_2$  equals the cumulative load per unit area of specimen), which causes additional deformation  $\Delta H_2$ , the void ratio at the end of consolidation can be calculated as

$$e_2 = e_1 - \frac{\Delta H_2}{H_2} \quad \text{Eq.10.38}$$

At this time,  $\sigma_2'$  effective stress, proceeding in a similar manner, one can obtain the void ratios at the end of the consolidation for all load increments.

The effective stress  $\sigma'$  and the corresponding void ratios ( $e$ ) at the end of consolidation are plotted on semi logarithmic graph paper. The typical shape of such a plot is shown in Figure 10.11

Fig. 10.11: Typical plot of  $e$  against  $\log \sigma'$



### 10.6.2 Evaluation of coefficient of consolidation from Oedometer test data

The coefficient of consolidation,  $C_v$ , in any stress range of interest, may be evaluated from its definition, however it can be evaluated by experimentally determining the parameters  $k$ ,  $a_v$  and  $e_0$  for the stress range under consideration.  $k$  may be got from a permeability test conducted on the oedometer sample itself, after complete consolidation under the particular stress increment  $a_v$  and  $e_0$  may be obtained from the oedometer test data, by plotting the  $e - \sigma$  curve. However, consolidation equation is rarely used for the determination of  $c_v$ . Instead,  $c_v$  is evaluated from the consolidation test data by the use of characteristics of the theoretical relationship between the time factor  $T$ , and the degree of consolidation,  $U$ . These methods are known as 'fitting methods', as one tries to fit in the characteristics of the theoretical curve with the experimental or laboratory curve. The more generally used fitting methods are the following:

- (a) The square root of time fitting method
- (b) The logarithm of time fitting method

These two methods will be presented in the following sub-sections.

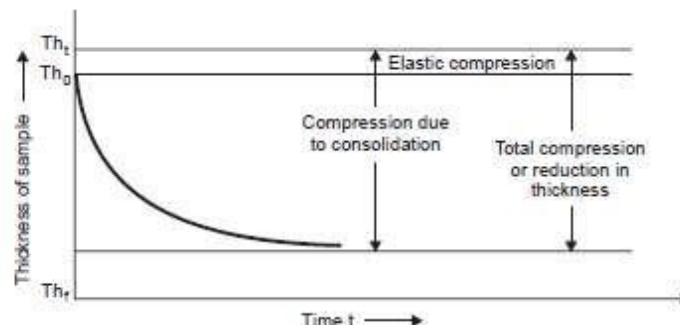


Fig. 10.12: Time versus reduction in sample thickness for a load-increment



### 10.6.3 The Square Root of Time Fitting Method

This method has been devised by D.W. Taylor (1948). The coefficient of consolidation is the soil property that controls the time-rate or speed of consolidation under a load-increment. The relation between the sample thickness and elapsed time since the application of the loading increment is obtainable from an oedometer test and is somewhat as shown in Figure 10.13 for a typical load-increment. This figure depicts change in sample thickness with time essentially due to consolidation only the elastic compression which occurs almost instantaneously on application of load increment is shown. The effect of prolonged compression that occurs after 100% dissipation of excess pore pressure is not shown or is ignored; this effect is known as ‘Secondary consolidation which is briefly presented in the following section. The curves of Figs.10.13 and the theoretical curves bear striking similarity; in fact, one should expect it if Terzaghi’s theory is to be valid for the phenomenon of consolidation. This similarity becomes more apparent if the curves are plotted with square root of time/time factor as the function, as shown in Fig. 10.13 (a) and (b). The theoretical curve on the square root plot is straight line up to about 60% consolidation with a gentle concave upwards curve thereafter. If another straight line, shown dotted, is drawn such that the abscissae of this line are 1.15 times those of the straight line portion of the theoretical curve, it can be shown to cut the theoretical curve at 90% consolidation. This may be established from the values of  $T$  at various values of  $U$  that is, the value of  $T$  at 90% consolidation is 1.15 times the abscissa of an extension of the straight line portion of the  $U$  versus  $T$  relation. This property is used for ‘fitting’ the theoretical curve to the laboratory curve. The laboratory curve shows a sudden initial compression, called ‘elastic compression’ which may be partly due to compression of gas in the pores. The corrected zero point at zero time is obtained by extending the straight line portion of the laboratory plot backward to meet the axis showing the sample thickness/dial gauge reading. The so-called ‘primary compression’ or ‘primary consolidation’ is reckoned from this corrected zero. A dashed line is constructed from the corrected zero such that its abscissae are 1.15 times those of the straight line portion of the laboratory plot. The intersection of the dashed line with the laboratory plot identifies the point representing 90% consolidation in the sample. The time corresponding to this can be read off from the laboratory plot. The point corresponding to 100% primary consolidation may be easily extrapolated on this plot.

The coefficient of consolidation,  $c_v$ , may be obtained from

$$c_v = \frac{T_{90}H^2}{t_{90}} \text{-----} \quad \text{Eq.10.39}$$

where  $t_{90}$  is read off from Fig. 10.7 (a)

$T_{90}$  is 0.848 from Terzaghi’s theory

$H$  is the drainage path, which may be taken as half the thickness of the sample for double drainage conditions,

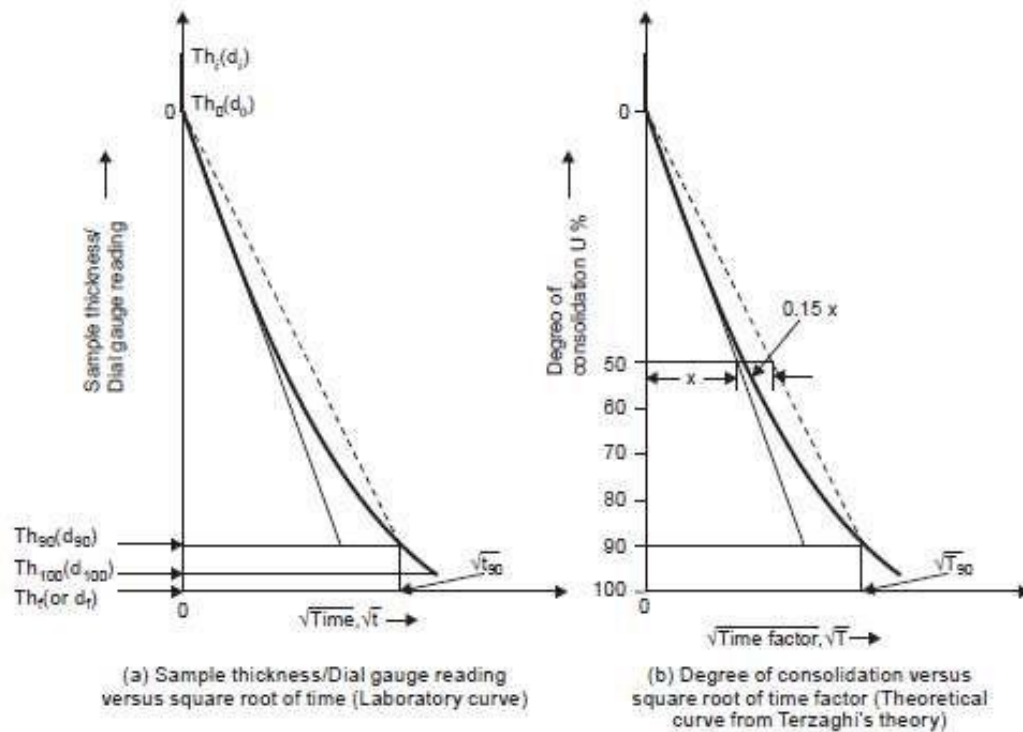


Fig. 10.7: Square root of time fitting method (After Taylor, 1948)

#### 10.6.4 The Logarithm of Time Fitting Method

This method was devised by A. Casagrande and R.E. Fadum (1939). The point corresponding to 100 per cent consolidation curve is plotted on a semi-logarithmic scale, with time factor on a logarithmic scale and degree of consolidation on arithmetic scale, the intersection of the tangent and asymptote is at the ordinate of 100% consolidation. A comparison of the theoretical and laboratory plots in this regard is shown in Figs. 10.8 (a) and (b).

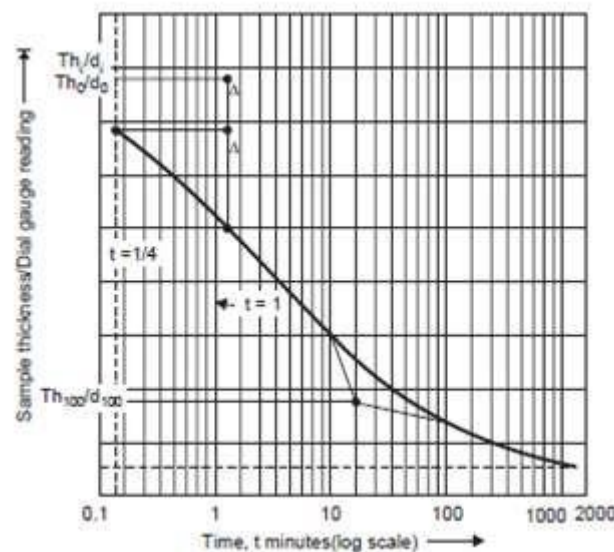


Fig. 10.8(a): Sample thickness/Dial gauge reading versus logarithm of time (Laboratory curve)

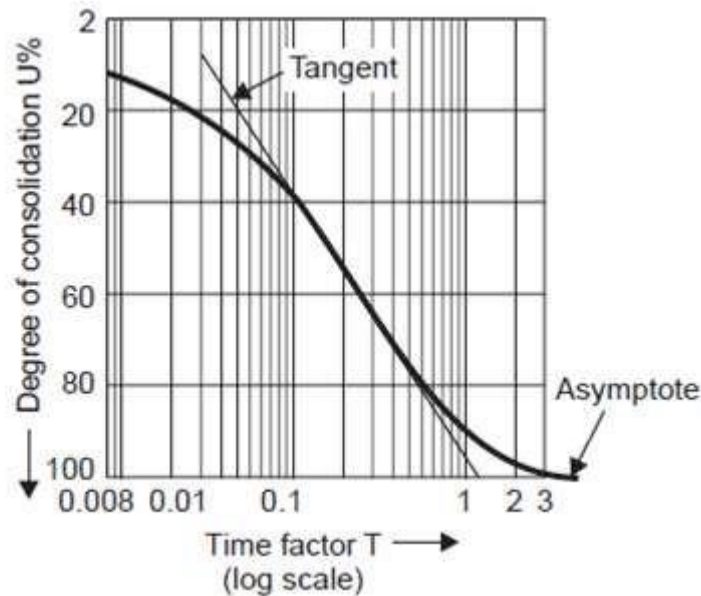


Fig. 10.8 (b) Logarithm of time fitting method (After A. Casagrande, 1939)

Since the early portion of the curve is known to approximate a parabola, the corrected zero point may be located as follows: The difference in ordinates between two points with times in the ratio of 4 to 1 is marked off; then a distance equal to this difference may be stepped off above the upper points to obtain the corrected zero point. This point may be checked by more trials, with different pairs of points on the curve. After the zero and 100% primary compression points are located, the point corresponding to 50% consolidation and its time may easily be obtained and the coefficient of consolidation computed from:

$$c_v = \frac{T_{50}H^2}{t_{50}} \text{-----} \quad \text{Eq.10.40}$$

Where  $t_{50}$  is read off from Fig. 10.8 (a)

$T_{50} = 0.197$  from Terzaghi's theory, and  $H$  is the drainage path as stated in the previous subsection.

## 10.7 SECONDARY CONSOLIDATION

The time-settlement curve for a cohesive soil has three distinct parts as illustrated in Figure 10.9. When the hydrostatic excess pressure is fully dissipated, no more consolidation should be expected. However, in practice, the decrease in void ratio continues, though very slowly, for a long time after this stage, called 'Primary Consolidation'. The effect or the phenomenon of continued consolidation after the complete dissipation of excess pore water pressure is termed 'Secondary Consolidation' and the resulting compression is called 'Secondary Compression'. During this stage, plastic readjustment of clay platelets takes place and other effects as well as colloidal-chemical processes and surface phenomena such as induced electrokinetic potentials occur. These are, by their very nature, very slow.

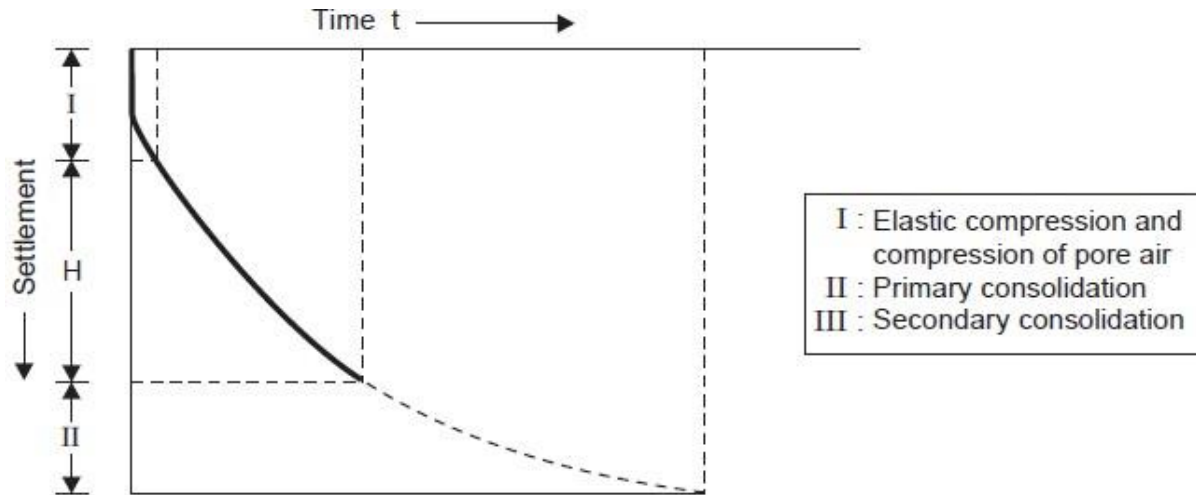


Fig.10.9: Time settlement curve for cohesive soil

Secondary consolidation is believed to come into play even in the range of primary consolidation, although its magnitude is small, because of the existence of a plastic lag right from the beginning of loading. However, it is almost impossible to separate this component from the primary compression. Since dissipation of excess pore pressure is not the criterion here, Terzaghi's theory is inapplicable to secondary consolidation. The fact that experimental time compression curves are in agreement with Terzaghi's theoretical curve only up to about 60% consolidation is, in itself, an indication of the manifestation of secondary consolidation even during the stage of primary consolidation.

Secondary consolidation of mineral soils is usually negligible but it may be considerable in the case of organic soils due to their colloidal nature. This may constitute a substantial part of total compression in the case of organic soils, micaceous soils, loosely deposited clays, etc. A possible disintegration of clay particles is also mentioned as one of the reasons for this phenomenon. Secondary compression is usually assumed to be proportional to the logarithm of time. Hence, the secondary compression can be identified on a plot of void ratio versus logarithm of time (Fig. 10.10).

Secondary compression appears as a straight line sloping downward or, in some cases, as a straight line followed by a second straight line with a flatter slope. The void ratio,  $e_f$ , at the end of primary consolidation can be found from the intersection of the backward extension of the secondary line with a tangent drawn to the curve of primary compression, as shown in the figure. The rate of secondary compression depends upon the increment of stress and the characteristics of the soil.

The equation for the rate of secondary compression may be approximated as follows:

$$\Delta e = -\alpha \cdot \log_{10}(t_2/t_1) \text{ -----} \quad \text{Eq.10.41}$$

Here,  $t_1$  is the time required for the primary compression to be virtually complete,  $t_2$  any later time, and  $\Delta e$  is the corresponding change in void ratio. This means that the secondary compression which occurs during the hydrodynamic phase is ignored, but the error is not probably serious.  $\alpha$  is a coefficient expressing the rate of secondary compression

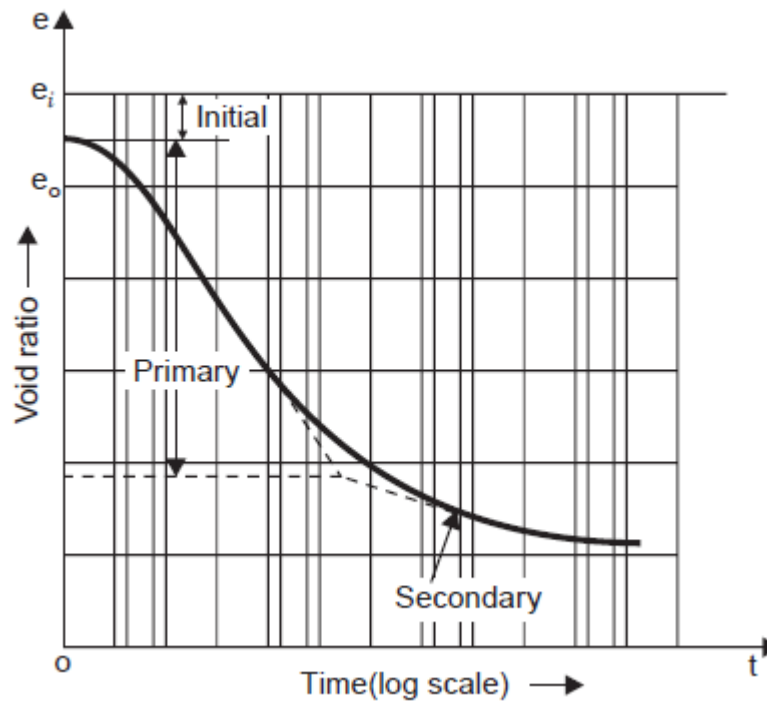


Fig.10.10: Voids ratio versus logarithmic of time

Another way of expressing the time-rate of secondary compression is through the 'coefficient of secondary compression',  $C_\alpha$ , in terms of strain or percentage of settlement as follows:

$$e = -C_\alpha \log_{10} \left( \frac{t_2}{t_1} \right) \text{-----} \quad \text{Eq.10.42}$$

In other words,  $C_\alpha$  may be taken to be the slope of the straight line representing the secondary compression on a plot of strain versus logarithm of time.

The relation between  $\alpha$  and  $C_\alpha$  is

$$C_\alpha = \frac{\alpha}{1+e} \text{-----} \quad \text{Eq.10.43}$$

Generally  $\alpha$  and  $C_\alpha$  increase with increasing stress

Some common values of  $C_\alpha$  are given below

Sl. No	Nature of Soil	$C_\alpha$ – Value
1	Over consolidated clays	0.0005 to 0.0015
2	Normally consolidated clays	0.005 to 0.030
3	Organic soils, peats	0.04 to 0.10

## 10.8 CALCULATION OF SETTLEMENT FROM ONE-DIMENSIONAL PRIMARY CONSOLIDATION

With the knowledge gained from the analysis of consolidation test results, we can now proceed to calculate the probable settlement caused by primary consolidation in the field, assuming one-dimensional consolidation.

Let us consider a saturated clay layer of thickness  $H$  and cross-sectional area  $A$  under an existing average effective overburden pressure. Because of an increase of effective pressure,  $\sigma'$ , let the primary settlement be  $S_c$ . Thus, the change in volume (Figure 10.11) can be given by

$$\Delta V = V_0 - V_1 = HA - (H - S_c)A \quad \text{Eq. 10.44}$$

$$\text{Hence } \Delta V = S_c A \quad \text{Eq. 10.43}$$

Where,  $V_0$  and  $V_1$  are the initial and final volumes, respectively. However, the change in the total volume is equal to the change in the volume of voids,  $\Delta V_V$ . Hence,

$$\Delta V = S_c A = V_{V0} - V_{V1} = \Delta V_V \quad \text{Eq. 10.44}$$

Where,  $V_{V0}$  and  $V_{V1}$  are the initial and final void volumes, respectively. From the definition of void ratio, it follows that

$$\Delta V = \Delta e V_S \quad \text{Eq. 10.45}$$

Where,  $\Delta e$  is the change of void ratio. But

$$V = \frac{V_0}{1+e_0} = \frac{AH}{1+e_0} \quad \text{Eq. 10.46}$$

Where,  $e_0$  is initial void ratio at volume  $V_0$ . Thus, from Eqs. (10.43) through (10.46),

$$\Delta V = S_c A = \Delta e V = \frac{AH}{1+e_0} \Delta e \quad \text{Eq. 10.47}$$

$$S_c = H \frac{\Delta e}{1+e_0} \quad \text{Eq. 10.48}$$

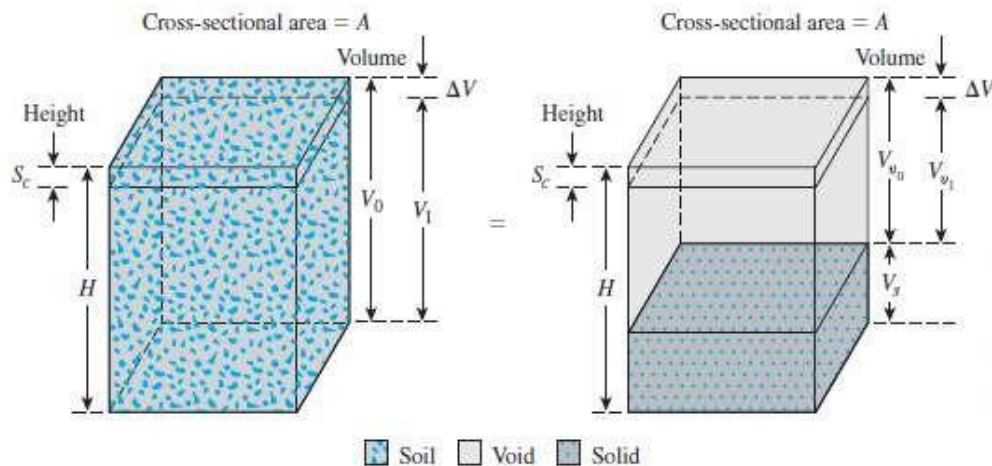


Fig.10.11: Settlement caused by one dimensional consolidation

For normally consolidated clays that exhibit a linear  $e-\log \sigma'$  relationship (see Figure 10.12),

$$\Delta e = c_c [\log_{10}(\sigma'_0 + \Delta \sigma') - \log_{10} \sigma'_0] \quad \text{Eq. 10.49}$$

Where,  $c_c$  is the slope of the  $e-\log \sigma'$  plot and is defined as the compression index. Substitution of Eq. (10.49) into Eq. (10.48) gives

$$S_C = \frac{c_c}{1+e_0} H \log_{10} \left( \frac{\sigma'_0}{\sigma'_0} \right) \text{-----} \quad \text{Eq.10.50}$$

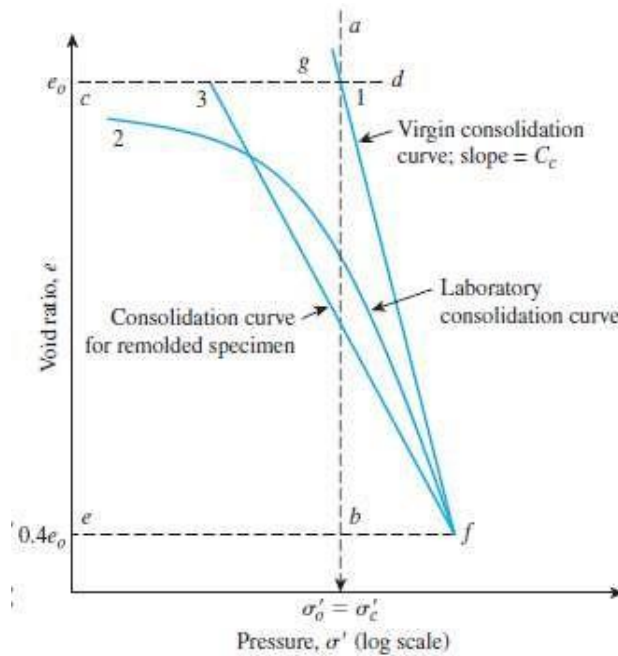


Fig.10.12: Consolidation characteristics of normally consolidated clays

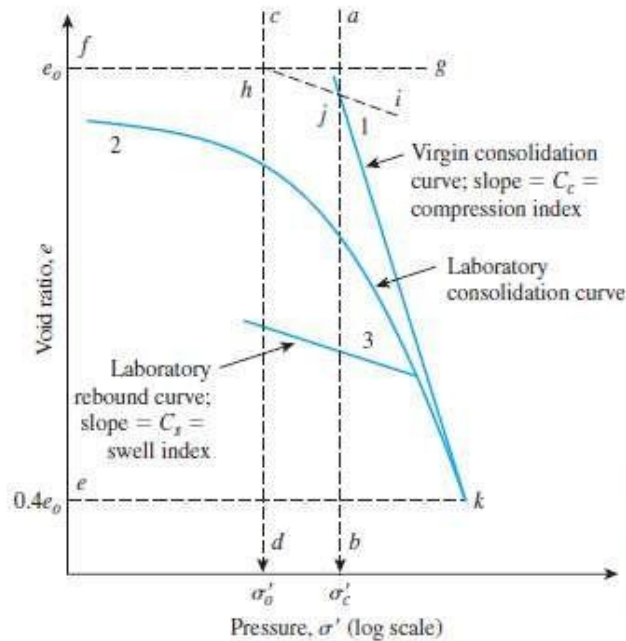


Fig.10.13: Consolidation characteristics of over consolidated clays

In over-consolidated clays (see Figure 10.13), for field  $e$ - $\log \sigma'$  variation will be along the line  $hj$ , the slope of which will be approximately equal to that for the laboratory rebound curve. The slope of the rebound curve  $C_s$  is referred to as the *swell index*; so

$$\Delta e = c_s [\log_{10}(\sigma'_0 + \Delta \sigma') - \log_{10} \sigma'_0] \text{-----} \quad \text{Eq.10.51}$$

Hence we obtain



$$S_c = \frac{c_s}{1+e_0} H \log_{10} \left( \frac{\sigma'_0}{\sigma'_c} \right) \text{-----} \quad \text{Eq.10.52}$$

If  $\sigma'_0 + \Delta\sigma' > \sigma'_c$  then

$$S_c = \frac{c_s}{1+e_0} H \log_{10} \left( \frac{\sigma'_c}{\sigma'_0} \right) + \frac{c}{1+e_0} H \log_{10} \left( \frac{\sigma'_0 + \Delta\sigma'}{\sigma'_c} \right) \text{-----} \quad \text{Eq.10.53}$$

### 10.9 CORRELATIONS FOR COMPRESSION INDEX ( $C_c$ )

The compression index for the calculation of field settlement caused by consolidation can be determined by graphic construction (as shown in Figure 10.13) after one obtains the laboratory test results for void ratio and pressure.

Skempton (1944) suggested the following empirical expression for the compression index for undisturbed clays:

$$C_c = 0.009(LL - 10\%) \text{-----} \quad \text{Eq.10.54}$$

Where LL is the liquid limit

Several other correlations for the compression index are also available. They have been developed by tests on various clays. Some of these correlations are given in Table 1.

Equation	Reference	Region of applicability
$C_c = 0.007(LL - 7)$	Skempton (1944)	Remolded clays
$C_c = 0.01w_N$		Chicago clays
$C_c = 1.15(e_0 - 0.27)$	Nishida (1956)	All clays
$C_c = 0.30(e_0 - 0.27)$	Hough (1957)	Inorganic cohesive soil: silt, silty clay, clay
$C_c = 0.0115w_N$		Organic soils, peats, organic silt, and clay
$C_c = 0.0046(LL - 9)$		Brazilian clays
$C_c = 0.75(e_0 - 0.5)$		Soils with low plasticity
$C_c = 0.208e_0 + 0.0083$		Chicago clays
$C_c = 0.156e_0 + 0.0107$		All clays

On the basis of observations on several natural clays, Rendon-Herrero (1983) gave the relationship for the compression index in the form

$$C_c = 0.14 G_s \left( \frac{1+e_0}{G_s} \right)^{2.38} \text{-----} \quad \text{Eq.10.55}$$

Nagaraj and Murty (1985) expressed the compression index as

$$C_c = 0.2343 \left( \frac{LL\%}{100} \right) G_s \text{-----} \quad \text{Eq.10.56}$$

More recently, Park and Koumoto (2004) expressed the compression index by the following relationship:

$$C_c = \frac{\eta_0}{371.747 - 4.275\eta_0} \text{-----} \quad \text{Eq.10.57}$$

where  $\eta_0 = \text{in situ porosity of the soil.}$

### 10.10 CORRELATIONS FOR SWELL INDEX ( $C_s$ )

The swell index is appreciably smaller in magnitude than the compression index and generally



can be determined from laboratory tests. In most cases,

$$c_s = \frac{1}{5} \text{ to } \frac{1}{10} c$$

Eq.10.58

The swell index was expressed by Nagaraj and Murty (1985) as

$$c_s = 0.0463 \left( \frac{LL\%}{100} \right) G_s \text{-----} \quad \text{Eq.10.59}$$

Based on the modified Cam clay model, Kulhawy and Mayne (1990) have shown that

$$c_s = \frac{PI}{370} \text{-----} \quad \text{Eq.10.60}$$

Where PI= Plasticity Index

**Example 1:** In a consolidation test the following results have been obtained. When the load was changed from 50 kN/m<sup>2</sup> to 100 kN/m<sup>2</sup>, the void ratio changed from 0.70 to 0.65. Determine the coefficient of volume decrease,  $m_v$  and the compression index,  $C_c$ .

**Sol:**

$$e_0 = 0.70 \quad \bar{\sigma}_0 = 50 \text{ kN/m}^2$$

$$e_1 = 0.65 \quad \bar{\sigma} = 100 \text{ kN/m}^2$$

$$\begin{aligned} \text{Coefficient of compressibility, } a_v &= \frac{\Delta e}{\Delta \bar{\sigma}}, \text{ ignoring sign.} \\ &= \frac{(0.70 - 0.65)}{(100 - 50)} \text{ m}^2/\text{kN} = 0.05/50 \text{ m}^2/\text{kN} = 0.001 \text{ m}^2/\text{kN}. \end{aligned}$$

Modulus of volume change, or coefficient of volume decrease,

$$\begin{aligned} m_v &= \frac{a_v}{(1 + e_0)} = \frac{0.001}{(1 + 0.70)} = \frac{0.001}{1.7} \text{ m}^2/\text{kN}. \\ &= 5.88 \times 10^{-4} \text{ m}^2/\text{kN} \end{aligned}$$

$$\begin{aligned} \text{Compression index, } C_c &= \frac{\Delta e}{\Delta (\log \bar{\sigma})} = \frac{(0.70 - 0.65)}{(\log_{10} 100 - \log_{10} 50)} \\ &= \frac{0.05}{\log_{10} \frac{100}{50}} = \frac{0.05}{\log_{10} 2} = \frac{0.050}{0.301} = 0.166. \end{aligned}$$

**Example 2:** A sand fill compacted to a bulk density of 18.84 kN/m<sup>3</sup> is to be placed on a compressible saturated marsh deposit 3.5 m thick. The height of the sand fill is to be 3 m. If the volume compressibility  $m_v$  of the deposit is  $7 \times 10^{-4} \text{ m}^2/\text{kN}$ , estimate the final settlement of the fill.

**Sol:**

Ht. of sand fill = 3 m

Bulk unit weight of fill = 18.84 kN/m<sup>3</sup>

Increment of the pressure on top of marsh deposit  $\Delta\bar{\sigma} = 3 \times 18.84$   
 $= 56.52 \text{ kN/m}^2$

Thickness of marsh deposit,  $H_0 = 3.5 \text{ m}$

Volume compressibility  $m_v = 7 \times 10^{-4} \text{ m}^2/\text{kN}$

Final settlement of the marsh deposit,  $\Delta H$

$$= m_v \cdot H_0 \cdot \Delta\bar{\sigma}$$

$$= 7 \times 10^{-4} \times 3500 \times 56.52 \text{ mm}$$

$$= 138.5 \text{ mm.}$$

**Example 3:** A layer of soft clay is 6 m thick and lies under a newly constructed building. The weight of sand overlying the clayey layer produces a pressure of 260 kN/m<sup>2</sup> and the new construction increases the pressure by 100 kN/m<sup>2</sup>. If the compression index is 0.5, compute the settlement. Water content is 40% and specific gravity of grains is 2.65.

Initial pressure,  $\bar{\sigma}_0 = 260 \text{ kN/m}^2$

Increment of pressure,  $\Delta\bar{\sigma} = 100 \text{ kN/m}^2$

**Sol:**

Thickness of clay layer,  $H = 6 \text{ m} = 600 \text{ cm}$ .

Compression index,  $C_c = 0.5$

Water content,  $w = 40\%$

Specific gravity of grains,  $G = 2.65$

Void ratio,  $e = wG$ , (since the soil is saturated)  $= 0.40 \times 2.65 = 1.06$

This is taken as the initial void ratio,  $e_0$ .

Consolidation settlement,

$$\begin{aligned} S &= \frac{H \cdot C_c}{(1 + e_0)} \log_{10} \left( \frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0} \right) \\ &= \frac{600 \times 0.5}{(1 + 1.06)} \log_{10} \left( \frac{260 + 100}{260} \right) \text{ cm} \\ &= \frac{300}{2.06} \log_{10} \left( \frac{360}{260} \right) \text{ cm} \\ &= 21.3 \text{ cm.} \end{aligned}$$

**Example 4 :** The settlement analysis (based on the assumption of the clay layer draining from top and bottom surfaces) for a proposed structure shows 2.5 cm of settlement in four years and an ultimate settlement of 10 cm. However, detailed sub-surface investigation reveals that there will be no drainage at the bottom. For this situation, determine the ultimate settlement and the time required for 2.5 cm settlement.

**Sol:**

The ultimate settlement is not affected by the nature of drainage, whether it is one-way or two-way.

Hence, the ultimate settlement = 10 cm.

However, the time-rate of settlement depends upon the nature of drainage.

Settlement in four years = 2.5 cm.

$$T = \frac{C_v t}{H^2}$$

$$U = \frac{2.5}{10.0} = 25\%$$

Since the settlement is the same,  $U\%$  is the same;  
hence, the time-factor is the same.

$$\therefore T/C_v = t/H^2 = \text{Constant.}$$

or 
$$\frac{t_2}{H_2^2} = \frac{t_1}{H_1^2},$$

$t_2$  and  $H_2$  referring to double drainage, and  $t_1$  and  $H_1$  referring to single drainage. The drainage path for single drainage is the thickness of the layer itself, while that for double drainage is half the thickness.

$$\therefore H_1 = 2H_2$$

$$\therefore \frac{t_2}{H_2^2} = \frac{t_1}{4H_2^2},$$

$$\therefore t_1 = 4t_2 = 4 \times 4 \text{ yrs} = 16 \text{ yrs.}$$

**Example 5 :** There is a bed of compressible clay of 4 m thickness with pervious sand on top and impervious rock at the bottom. In a consolidation test on an undisturbed specimen of clay from this deposit 90% settlement was reached in 4 hours. The specimen was 20 mm thick. Estimate the time in years for the building founded over this deposit to reach 90% of its final settlement.

**Sol:**

This is a case of one-way drainage in the field.

$\therefore$  Drainage path for the field deposit,  $H_f = 4 \text{ m} = 4000 \text{ mm}$ . In the laboratory consolidation test, commonly it is a case of two-way drainage.

$\therefore$  Drainage path for the laboratory sample,  $H_l = 20/2 = 10 \text{ mm}$

Time for 90% settlement of laboratory sample = 4 hrs.

Time factor for 90% settlement,  $T_{90} = 0.848$

$$\therefore T_{90} = \frac{C_v t_{90_f}}{H_f^2} = \frac{C_v t_{90_l}}{H_l^2}$$

or 
$$\frac{t_{90_f}}{H_f^2} = \frac{t_{90_l}}{H_l^2}$$

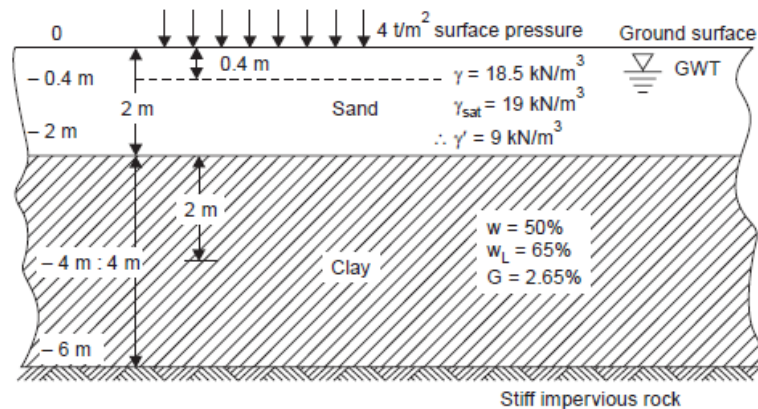
$$\begin{aligned} \therefore t_{90_f} &= \frac{t_{90_l}}{H_l} \times H_f = \frac{4 \times (4000)^2}{10^2} \text{ hrs} \\ &= \frac{4 \times 400}{24 \times 365} \text{ years} \\ &\approx 73 \text{ years.} \end{aligned}$$



**Example 6:** (a) The soil profile at a building site consists of dense sand up to 2 m depth, normally loaded soft clay from 2 m to 6 m depth, and stiff impervious rock below 6 m depth. The ground-water table is at 0.40 m depth below ground level. The sand has a density of  $18.5 \text{ kN/m}^3$  above water table and  $19 \text{ kN/m}^3$  below it. For the clay, natural water content is 50%, liquid limit is 65% and grain specific gravity is 2.65. Calculate the probable ultimate settlement resulting from a uniformly distributed surface load of  $40 \text{ kN/m}^2$  applied over an extensive area of the site.

(b) In a laboratory consolidation test with porous discs on either side of the soil sample, the 25 mm thick sample took 81 minutes for 90% primary compression. Calculate the value of coefficient of consolidation for the sample.

(a) The soil profile is as shown in Fig. 1:



**Fig.1: Soil Profile at Building Site**

**Sol:**

For the clay stratum:

$$w = 50\%$$

$$G = 2.65$$

Since it is saturated,  $e = w \cdot G = 0.50 \times 2.65 = 1.325$

This is the initial void ratio,  $e_0$ .

$$\gamma_{\text{sat}} = \frac{(G + e)}{(1 + e)} \gamma_w = \frac{(2.65 + 1.325)}{(1 + 1.325)} \times 10 \text{ kN/m}^3 = \frac{3.975 \times 10}{2.325} \text{ kN/m}^3$$

$$= 17.1 \text{ kN/m}^3$$

$$\gamma = (\gamma_{\text{sat}} - \gamma_w) = 7.1 \text{ kN/m}^3$$

Initial effective overburden pressure at the middle of the clay layer:

$$\bar{\sigma}_0 = (0.4 \times 18.5 + 1.6 \times 9.0 + 2 \times 7.1) \text{ t/m}^2$$

$$= 36 \text{ kN/m}^2$$

The compression index,  $C_c$  may be taken as:

$$C_c = 0.009 (w_L - 10)$$

$$\therefore C_c = 0.009 (65 - 10) = 0.495$$

The consolidation settlement,  $S$ , is given by:

$$\begin{aligned} S &= \frac{H \cdot C_c}{(1 + e_0)} \log_{10} \left( \frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0} \right) \\ &= \frac{400 \times 0.495}{(1 + 1.325)} \log_{10} \frac{(36 + 40)}{36} \text{ cm} \\ &= 27.64 \text{ cm.} \end{aligned}$$

Let us assume that the applied surface pressure of  $4.0 \text{ t/m}^2$  gets transmitted to the middle of the clay layer undiminished.

$$\therefore \Delta \bar{\sigma} = 40 \text{ kN/m}^2$$

(b) Thickness of the laboratory sample = 25 mm.

Since it is two-way drainage with porous discs on either side, the drainage path,

$$H = 25/2 = 12.5 \text{ mm.}$$

Time for 90% primary compression,  $t_{90} = 81$  minutes.

Time factor,  $T_{90}$ , for  $U = 90\%$  is known to be 0.848.

$$\begin{aligned} \text{(Alternatively, } T &= -0.9332 \log_{10} (1 - U) - 0.0851 \\ &= -0.9332 \log_{10} 0.10 - 0.0851 = 0.9332 - 0.0851 = 0.8481 \end{aligned}$$

$$\therefore T_{90} = \frac{C_v t_{90}}{H^2}$$

Coefficient of consolidation

$$\begin{aligned} C_v &= \frac{T_{90} \cdot H^2}{t_{90}} = \frac{0.848 \times (125)^2}{81} \text{ cm}^2/\text{min.} \\ &= \frac{0.848 \times (125)^2}{81 \times 60} \text{ cm}^2/\text{s} \\ &= 2.726 \times 10^{-4} \text{ cm}^2/\text{s.} \end{aligned}$$

### Example 7:

A sample, 75 mm in diameter and 20 mm high, taken from a clay layer 10 m thick, was tested in an oedometer with drainage at the upper and lower boundaries. It took the laboratory sample 15 minutes to reach 50% consolidation.

- If the clay layer in the field has the same drainage condition as the laboratory sample, calculate how long it will take the 10-m clay layer to achieve 50% and 90% consolidation.
- How much more time would it take the 10-m clay layer to achieve 50% consolidation if drainage existed on only one boundary?

**Sol:**

(a) We proceed as follows:

**Step 1:** Calculate the drainage path.

$$(H_{dr})_{lab} = \frac{20}{2} = 10 \text{ mm} = 0.01 \text{ m}; \quad (H_{dr})_{field} = \frac{10}{2} = 5 \text{ m}$$

**Step 2:** Calculate the field time using Equation (9.48).

$$t_{field} = \frac{t_{lab}(H_{dr}^2)_{field}}{(H_{dr}^2)_{lab}} = \frac{15 \times 5^2}{0.01^2} = 375 \times 10^4 \text{ min} = 7.13 \text{ years}$$

(b) We proceed as follows:

**Step 1:** Calculate the drainage path.

$$(H_{dr})_{lab} = \frac{20}{2} = 10 \text{ mm} = 0.01 \text{ m}; \quad (H_{dr})_{field} = 10 \text{ m}$$

**Step 2:** Calculate field time using Equation (9.48).

$$t_{field} = \frac{t_{lab}(H_{dr}^2)_{field}}{(H_{dr}^2)_{lab}} = \frac{15 \times 10^2}{0.01} = 15 \times 10^6 \text{ min} = 28.54 \text{ years}$$

### Example 8:

At a vertical stress of 200 kPa, the void ratio of a saturated soil sample tested in an oedometer is 1.52 and lies on the normal consolidation line. An increment of vertical stress of 150 kPa compresses the sample to a void ratio of 1.43.

- Determine the compression index,  $C_c$ , of the soil.
- The sample was unloaded to a vertical stress of 200 kPa, and the void ratio increased to 1.45. Determine the slope of the recompression index,  $C_r$ .
- What is the overconsolidation ratio of the soil at stage (b)?
- If the soil were reloaded to a vertical stress of 500 kPa, what void ratio would be attained?

**Sol:**

Draw a sketch of the soil response on a plot of vertical effective stress (log scale) versus void ratio. Use this sketch to answer the various questions.

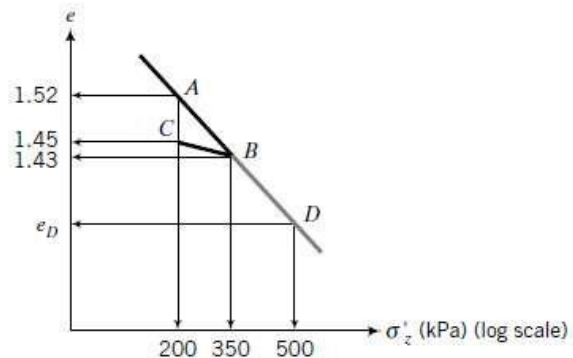
**Step 1:** Determine  $C_c$ .

$C_c$  is the slope  $AB$  shown in Figure

$$C_c = \frac{-(1.43 - 1.52)}{\log(350/200)} = 0.37$$

**Step 2:** Determine  $C_r$ .

$C_r$  is the slope of  $BC$  in Figure





$$C_r = \frac{-(1.43 - 1.45)}{\log(350/200)} = 0.08$$

**Step 3:** Determine the overconsolidation ratio.

Past maximum vertical effective stress:  $\sigma'_{zc} = 350$  kPa

Current vertical effective stress:  $\sigma'_z = 200$  kPa

$$\text{OCR} = \frac{\sigma'_{zc}}{\sigma'_z} = \frac{350}{200} = 1.75$$

**Step 4:** Calculate the void ratio at 500 kPa.

The void ratio at 500 kPa is the void ratio at *D* on the normal consolidation line

$$e_D = e_B - C_c \log\left(\frac{500}{350}\right) = 1.43 - 0.37 \log 1.43 = 1.37$$

### Example 9:

The following readings were taken for an increment of vertical stress of 20 kPa in an oedometer test on a saturated clay sample 75 mm in diameter and 20 mm thick. Drainage was permitted from the top and bottom boundaries.

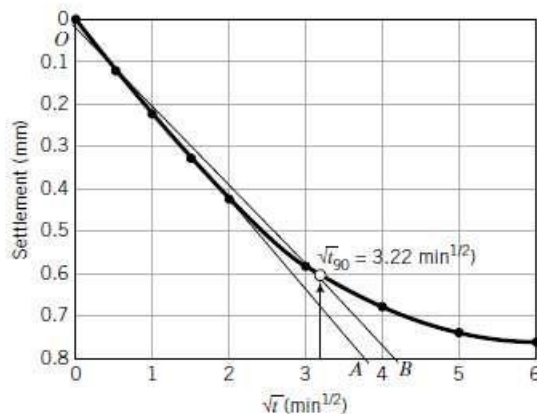
Time (min)	0.25	1	2.25	4	9	16	25	36	24 hours
$\Delta H$ (mm)	0.12	0.23	0.33	0.43	0.59	0.68	0.74	0.76	0.89

Determine the coefficient of consolidation using the root time method.

**Sol:**

Plot the data in a graph of displacement reading versus  $\sqrt{\text{time}}$  and follow the procedures

**Step 1:** Make a plot of settlement (decrease in thickness) versus  $\sqrt{\text{time}}$ , as shown in Figure



**FIGURE**

**Step 2:** Follow the procedures outlined in Section 9.7.2.1 to find  $t_{90}$ .

From Figure

$$\sqrt{t_{90}} = 3.22 \text{ min}^{1/2}; \quad t_{90} = 10.4 \text{ min}$$

**Step 3:** Calculate  $C_v$

$$C_v = \frac{0.848 H_{dr}^2}{t_{90}}$$

where  $H_{dr}$  is the length of the drainage path. The current height is  $20 - 0.89 = 19.1$  mm.

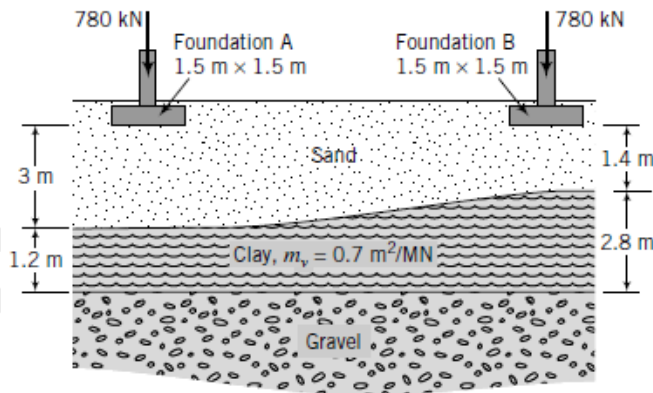
$$H_{dr} = \frac{H_o + H_f}{4} = \frac{20 + 19.1}{4} = 9.8 \text{ mm}$$

$$\therefore C_v = \frac{0.848 \times 9.8^2}{10.4} = 7.8 \text{ mm}^2/\text{min}$$

### Example 10

The foundations supporting two columns of a building are shown in Figure . An extensive soil investigation was not carried out, and it was assumed in the design of the foundations that the clay layer had a uniform thickness of 1.2 m. Two years after construction the building settled, with a differential settlement of 10 mm. Walls of the building began to crack. The doors have not jammed, but by measuring the out-of-vertical distance of the doors,

it is estimated that they would become jammed if the differential settlement exceeded 24 mm. A subsequent soil investigation showed that the thickness of the clay layer was not uniform but varies, as shown in Figure . The owners would like to get an estimate of the expected total differential settlement and how long it will take before the doors become jammed.



**Sol:**

**Step 1:** Calculate the vertical stress increase at the center of the clay layer under each foundation. Let's use the approximate method.

$$\Delta\sigma_z = \frac{P}{(B + z)(L + z)}$$

$$(\Delta\sigma_z)_A = \frac{780}{(1.5 + 3.6)(1.5 + 3.6)} = 30 \text{ kPa}$$

$$(\Delta\sigma_z)_B = \frac{780}{(1.5 + 2.8)(1.5 + 2.8)} = 42.2 \text{ kPa}$$

**Step 2:** Calculate the primary consolidation settlement.

Use  $\rho_{pc} = H_o m_v \Delta \sigma$  to calculate the primary consolidation settlement.

$$(\rho_{pc})_A = 1.2 \times 0.7 \times 10^{-3} \times 30 = 25.2 \times 10^{-3} \text{ m} = 25.2 \text{ mm}$$

$$(\rho_{pc})_B = 2.8 \times 0.7 \times 10^{-3} \times 42.2 = 82.7 \times 10^{-3} \text{ m} = 82.7 \text{ mm}$$

**Step 3:** Calculate the differential settlement.

$$\text{Differential settlement: } \delta = 82.7 - 25.2 = 57.5 \text{ mm}$$

**Step 4:** Calculate the time for 24-mm differential settlement to occur.

$$\text{Current differential settlement: } \delta_c = 10 \text{ mm}$$

$$\text{Degree of consolidation: } U = \frac{\delta_c}{\delta} = \frac{10}{57.5} = 0.17$$

$$T_v = \frac{4}{\pi} U^2 = \frac{4}{\pi} \times 0.17^2 = 0.037$$

$$C_v = \frac{T_v H_{dr}^2}{t} = \frac{0.037 \times (2.8/2)^2}{2} = 0.036 \text{ m}^2/\text{yr}$$

$$\text{For 24-mm differential settlement: } U = \frac{24}{57.5} = 0.42, \quad T_v = \frac{4}{\pi} \times 0.42^2 = 0.225$$

$$t = \frac{T_v H_{dr}^2}{C_v} = \frac{0.225 \times (2.8/2)^2}{0.036} = 12.25 \text{ years}$$

Therefore, in the next 10.25 years, the total differential settlement would be 24 mm.

### Example 11:

The following compression readings were obtained in an oedometer test on a specimen of saturated clay ( $G_s = 2.73$ ):

Pressure (kN/m <sup>2</sup> )	0	54	107	214	429	858	1716	3432	0
Dial gauge after 24 h (mm)	5.000	4.747	4.493	4.108	3.449	2.608	1.676	0.737	1.480

The initial thickness of the specimen was 19.0 mm and at the end of the test the water content was 19.8%. Plot the  $e$ - $\log \sigma'$  curve and determine the preconsolidation pressure. Determine the values of  $m_v$  for the stress increments 100–200 and 1000–1500 kN/m<sup>2</sup>. What is the value of  $C_c$  for the latter increment?

**Sol:**

$$\text{Void ratio at end of test} = e_1 = w_1 G_s = 0.198 \times 2.73 = 0.541$$

$$\text{Void ratio at start of test} = e_0 = e_1 + \Delta e$$

Now

$$\frac{\Delta e}{\Delta H} = \frac{1 + e_0}{H_0} = \frac{1 + e_1 + \Delta e}{H_0}$$

$$\text{Hence } \frac{\Delta e}{3.520} = \frac{1.541 + \Delta e}{19.0}$$

$$\text{OR } \Delta e = 0.350 \text{ and } e_0 = 0.541 + 0.350 = 0.891$$

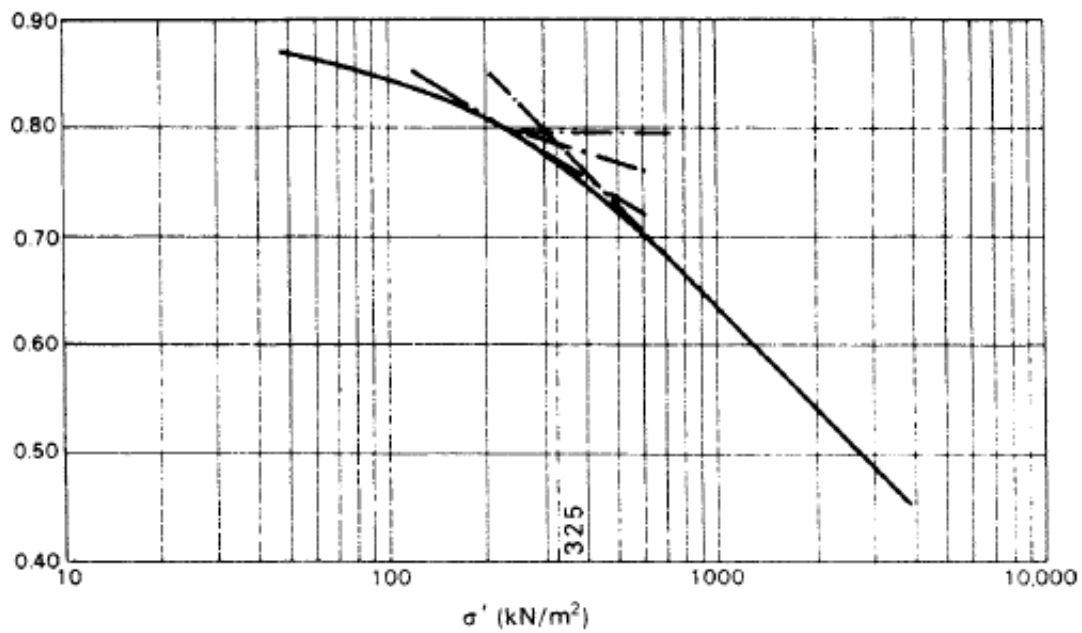
In general, the relationship between  $\Delta e$  and  $\Delta H$  is given by

$$\frac{\Delta e}{\Delta H} = \frac{1.891}{19.0}$$

i.e.  $\Delta e = 0.0996 \Delta H$ , and can be used to obtain the void ratio at the end of each increment period (see Table 1). The  $e$ - $\log \sigma'$  curve using these values is shown in Figure. Using Casagrande's construction, the value of the preconsolidation pressure is  $325 \text{ kN/m}^2$ .

Table 1:

Pressure (kN/m <sup>2</sup> )	$\Delta H$ (mm)	$\Delta e$	$e$
0	0	0	0.891
54	0.253	0.025	0.866
107	0.507	0.050	0.841
214	0.892	0.089	0.802
429	1.551	0.154	0.737
858	2.392	0.238	0.653
1716	3.324	0.331	0.560
3432	4.263	0.424	0.467
0	3.520	0.350	0.541



For  $\sigma'_0 = 100 \text{ kN/m}^2$  and  $\sigma'_1 = 200 \text{ kN/m}^2$ ,

$$e_0 = 0.845 \quad \text{and} \quad e_1 = 0.808$$

and therefore

$$m_v = \frac{1}{1.845} \times \frac{0.037}{100} = 2.0 \times 10^{-4} \text{ m}^2/\text{kN} = 0.20 \text{ m}^2/\text{MN}$$

For  $\sigma'_0 = 1000 \text{ kN/m}^2$  and  $\sigma'_1 = 1500 \text{ kN/m}^2$ ,

$$e_0 = 0.632 \quad \text{and} \quad e_1 = 0.577$$

and therefore

$$m_v = \frac{1}{1.632} \times \frac{0.055}{500} = 6.7 \times 10^{-5} \text{ m}^2/\text{kN} = 0.067 \text{ m}^2/\text{MN}$$

and

$$C_c = \frac{0.632 - 0.577}{\log(1500/1000)} = \frac{0.055}{0.176} = 0.31$$

Note that  $C_c$  will be the same for any stress range on the linear part of the  $e$ - $\log \sigma'$  curve;  $m_v$  will vary according to the stress range, even for ranges on the linear part of the curve.



## 11.0 SHEAR STRENGTH OF SOILS

### INTRODUCTION

Shearing Strength' of a soil is perhaps the most important of its engineering properties. This is because all stability analyses in the field of geotechnical engineering, whether they relate to foundation, slopes of cuts or earth dams, involve a basic knowledge of this engineering property of the soil. 'Shearing strength' or merely 'Shear strength' may be defined as the resistance to shearing stresses and a consequent tendency for shear deformation.

Shearing strength of a soil is the most difficult to comprehend in view of the multitude of factors known to affect it. A lot of maturity and skill may be required on the part of the engineer in interpreting the results of the laboratory tests for application to the conditions in the field.

Basically speaking, a soil derives its shearing strength from the following:

- (1) Resistance due to the interlocking of particles.
- (2) Frictional resistance between the individual soil grains, which may be sliding friction, rolling friction, or both.
- (3) Adhesion between soil particles or 'cohesion'.

Granular soils of sands may derive their shear strength from the first two sources, while cohesive soils or clays may derive their shear strength from the second and third sources.

Highly plastic clays, however, may exhibit the third source alone for their shearing strength.

Most natural soil deposits are partly cohesive and partly granular and as such, may fall into the second of the three categories just mentioned, from the point of view of shearing strength. The shear strength of a soil cannot be tabulated in codes of practice since a soil can significantly exhibit different shear strengths under different field and engineering conditions.

### 11.1 *Internal Friction within Granular Soil Masses*

In granular or cohesionless soil masses, the resistance to sliding on any plane through the point within the mass is similar to that discussed in the previous sub-section; the friction angle in this case is called the 'angle of internal friction'. However, the frictional resistance in granular soil masses is rather more complex than that between solid bodies, since the nature of the resistance is partly sliding friction and partly rolling friction. Further, a phenomenon known as 'interlocking' is also supposed to contribute to the shearing resistance of such soil masses, as part of the frictional resistance.

The angle of internal friction, which is a limiting angle of obliquity and hence the primary criterion for slip or failure to occur on a certain plane, varies appreciably for a given sand with the density index, since the degree of interlocking is known to be directly dependent upon the density. This angle also varies somewhat with the normal stress. However, the angle of internal friction is mostly considered constant, since it is almost so for a given sand at a given density.

Since failure or slip within a soil mass cannot be restricted to any specific plane, it is necessary to understand the relationships that exist between the stresses on different planes passing through a point, as a prerequisite for further consideration of shearing strength of soils.

### 11.2 PRINCIPAL PLANES AND PRINCIPAL STRESSES—MOHR'S CIRCLE

At a point in a stressed material, every plane will be subjected, in general, to a normal or direct stress and a shearing stress. In the field of geotechnical engineering, compressive direct stresses are usually considered positive, while tensile stresses are considered negative. A 'Principal plane' is defined as a plane on which the stress is wholly normal, or one which does not carry shearing stress. From mechanics, it is known that there exists three principal planes at any point in a stressed material. The normal stresses acting on these principal planes are known as the 'principal stresses'. The three principal planes are to be mutually perpendicular. In the order of decreasing magnitude the principal stresses are designated the 'major principal stress', the 'intermediate principal stress' and the 'minor principal stress', the corresponding principal planes being designated exactly in the same manner. It can be engineering by two-dimensional analysis, the intermediate principal stress being commonly ignored.

Let us consider an element of soil whose sides are chosen as the principal planes, the major and the minor, as shown in Fig. 11.1(a):

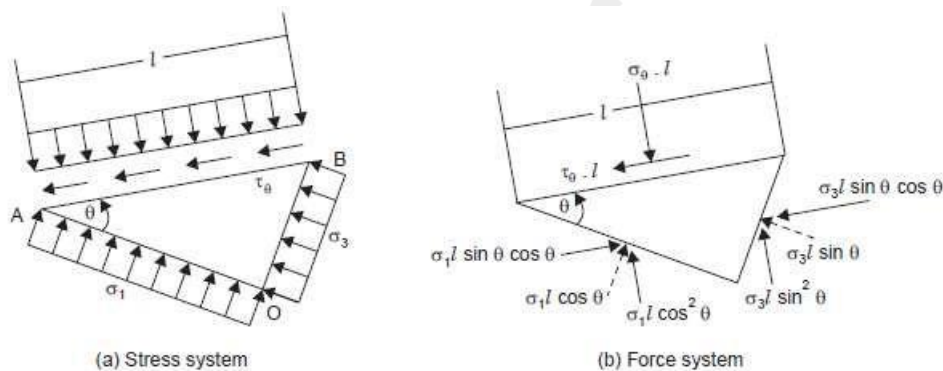


Fig. 11.1: Stresses on a plane inclined to the principal planes

Let  $O$  be any point in the stressed medium and  $OA$  and  $OB$  be the major and minor principal planes, with the corresponding principal stresses  $\sigma_1$  and  $\sigma_3$ . The plane of the figure is the intermediate principal plane. Let it be required to determine the stress conditions on a plane normal to the figure, and inclined at an angle  $\theta$  to the major principal plane, considered positive when measured counter-clockwise. If the stress conditions are uniform, the size of the element is immaterial. If the stresses are varying, the element must be infinitesimal in size, so that the variation of stress along a side needed to be considered.

Let us consider the element to be of unit thickness perpendicular to the plane of the figure,  $AB$  being  $l$ . The forces on the sides of the element are shown dotted and their components parallel and perpendicular to  $AB$  are shown by full lines. Considering the equilibrium of the element and resolving all forces in the directions parallel and perpendicular to  $AB$ , the following equations may be obtained:

$$\sigma_\theta = \sigma_1 \cos^2 \theta + \sigma_3 \sin^2 \theta = \sigma_3 (\sigma_1 - \sigma_3) \cos^2 \theta$$





These are the same as in Eqs. 11.1 and 11.2, which prove our statements. In the special case where the major and minor principal planes are vertical and horizontal respectively, or vice-versa, the origin of planes will be  $D$  or  $E$ , as the case may be. In other words, it will lie on the  $\sigma$ -axis. A few important basic facts and relationships may be directly obtained from the Mohr's circle:

1. The only planes free from shear are the given sides of the element which are the principal planes. The stresses on these are the greatest and smallest normal stresses.

2. The maximum or principal shearing stress is equal to the radius of the Mohr's circle, and it occurs on planes inclined at  $45^\circ$  to the principal planes.

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \text{-----} \quad \text{Eq.11.3}$$

3. The normal stresses on planes of maximum shear are equal to each other and is equal to half the sum of the principal stresses.

$$\sigma_c = \frac{(\sigma_1 + \sigma_3)}{2} \text{-----} \quad \text{Eq.11.4}$$

4. Shearing stresses on planes at right angles to each other are numerically equal and are of an opposite sign. These are called conjugate shearing stresses.

5. The sum of the normal stresses on mutually perpendicular planes is a constant ( $MG' + MG = 2MF = \sigma_1 + \sigma_3$ ). If we designate the normal stress on a plane perpendicular to the plane on which it is  $\sigma_\theta$  as  $\sigma'_\theta$

$$\sigma_\theta + \sigma'_\theta = \sigma_1 + \sigma_3 \text{-----} \quad \text{Eq.11.5}$$

Of the two stresses  $\sigma_\theta$  and  $\sigma'_\theta$  the one which makes the smaller angle with  $\sigma_1$  is the greater of the two.

6. The resultant stress,  $\sigma_r$ , on any plane is

$$\sigma_r = \sqrt{\sigma_\theta^2 + \tau_\theta^2} \text{-----} \quad \text{Eq.11.6}$$

and has an obliquity,  $\beta$ , which is equal to

$$\beta = \tan^{-1} \left( \frac{\tau_\theta}{\sigma_\theta} \right) \text{-----} \quad \text{Eq.11.7}$$

7. Stresses on conjugate planes, that is, planes which are equally inclined in different directions with respect to a principal plane are equal. (This is indicated by the co-ordinates of  $C$  and  $C1$  in Fig. 11.2).

8. When the principal stresses are equal to each other, the radius of the Mohr's circle becomes zero, which means that shear stresses vanish on all planes. Such a point is called an isotropic point.

9. The maximum angle of obliquity,  $\beta_m$ , occurs on a plane inclined at

$$\theta_{cr} = \frac{45^\circ + \beta_m}{2} \text{-----} \quad \text{Eq.11.8}$$

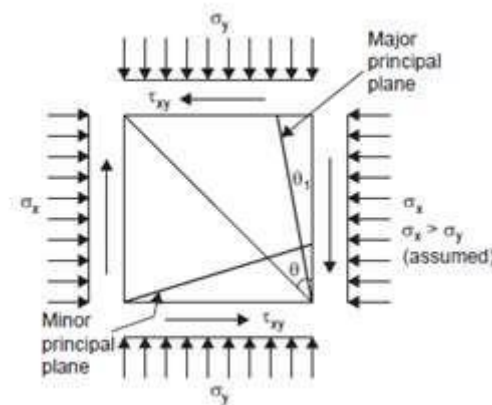
This may be obtained by drawing a line which passes through the origin and is tangential to the Mohr's circle. The co-ordinates of the point of tangency are the stresses on the plane of maximum obliquity; the shear stress on this plane is obviously less than the principal or maximum shear stress. On the plane of principal shear the obliquity is slightly smaller than  $\beta_m$ .

It is the plane of maximum obliquity which is most liable to failure and not the plane of maximum shear, since the criterion of slip is limiting obliquity. When  $\beta_m$  approaches and equals the angle of internal friction,  $\phi$ , of the soil, failure will become incipient. Mohr's circle affords an easy means of obtaining all important relationships. The following are a few such relationships

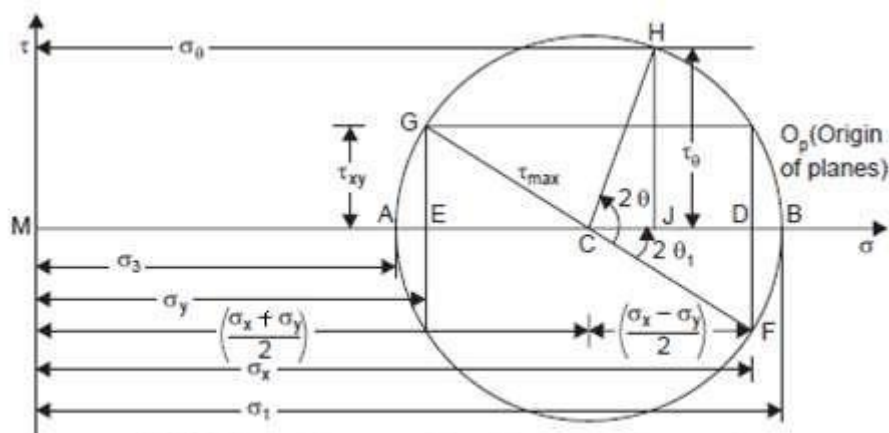
$$\sin \beta_m = \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 + \sigma_3)} \quad \text{Eq.11.9}$$

$$\frac{\sigma_1}{\sigma_3} = \left( \frac{1 + \sin \beta_m}{1 - \sin \beta_m} \right) \quad \text{Eq.11.10}$$

In case the normal and shearing stresses on two mutually perpendicular planes are known, the principal planes and principal stresses may be determined with the aid of the Mohr's circle diagram, as shown in Fig. 11.3(a). The shearing stresses on two mutually perpendicular planes are equal in magnitude by the principle of complementary shear



(a) General two-dimensional stress system



(b) Mohr's circle for general two-dimensional stress system

(b) Mohr's circle for general two-dimensional stress system

Fig. 11.3: Determination of principal planes and principal stresses from Mohr's circle  
Figure 11.3 (a) shows an element subjected to a general two-dimensional stress system, normal stresses  $\sigma_x$  and  $\sigma_y$  on mutually perpendicular planes and shear stresses  $\tau_{xy}$  on these planes, as

indicated. Fig. 11.3 (b) shows the corresponding Mohr's circle, the construction of which is obvious.

From a consideration of the equilibrium of a portion of the element, the normal and shearing stress components,  $\sigma_\theta$  and  $\tau_\theta$ , respectively, on a plane inclined at an angle  $\theta$ , measured counter-clockwise with respect to the plane on which  $\sigma_x$  acts, may be obtained as follows:

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \left( \frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta \quad \text{Eq.11.11}$$

$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta \quad \text{Eq.11.12}$$

Squaring and adding these Eqs, we obtain

$$\left[ \sigma_\theta - \frac{\sigma_x + \sigma_y}{2} \right]^2 + \tau_\theta^2 = \left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2 \quad \text{Eq.11.13}$$

This represents a circle with centre  $\frac{\sigma_x + \sigma_y}{2}$ , 0 and radius  $\sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$

Once the Mohr's circle is constructed, the principal stresses  $\sigma_1$  and  $\sigma_3$ , and the orientation of the principal planes may be obtained from the diagram. The shearing stress is to be plotted upward or downward according as it is positive or negative. It is common to take a shear stress which tends to rotate the element counter-clockwise, positive.

It may be noted that the same Mohr's circle and hence the same principal stresses are obtained, irrespective of how the shear stresses are plotted. (The centre of the Mohr's circle, C, is the mid-point of DE, with the co-ordinates and 0; the radius of the circle is CG), the co-ordinates of G being  $\sigma_y$  and  $\tau_{xy}$

The following relationships are also easily obtained

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2} \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + 4\tau_{xy}^2} \quad \text{Eq.11.14}$$

$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2} \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + 4\tau_{xy}^2} \quad \text{Eq.11.15}$$

The direction of principal stresses is expressed as given below

$$\tan(2\theta)_{1,3} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad \text{Eq.11.16}$$

$$\tau_{\max} = \frac{1}{2} \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + 4\tau_{xy}^2} \quad \text{Eq.11.17}$$

Invariably, the vertical stress will be the major principal stress and the horizontal one the minor principal stress in geotechnical engineering situations.

### 11.2.1 Mohr's Strength Theory

We have seen that the shearing stress may be expressed as  $\tau = \sigma \tan \beta$  on any plane, where  $\beta$  is the angle of obliquity. If the obliquity angle is the maximum or has limiting value  $\phi$ , the shearing stress

is also at its limiting value and it is called the shearing strength,  $s$ . For a cohesionless soil the shearing strength may be expressed as:

$$s = \sigma \tan \varphi \dots$$

Eq.11.18

If the angle of internal friction  $\varphi$  is assumed to be a constant, the shearing strength may be represented by a pair of straight lines at inclinations of  $+\varphi$  and  $-\varphi$  with the  $\sigma$ -axis and passing

through the origin of the Mohr's circle diagram. A line of this type is called a Mohr envelope. The Mohr envelopes for a cohesionless soil, as shown in Fig. 11.4, are the straight lines  $OA$  and  $OA'$ .

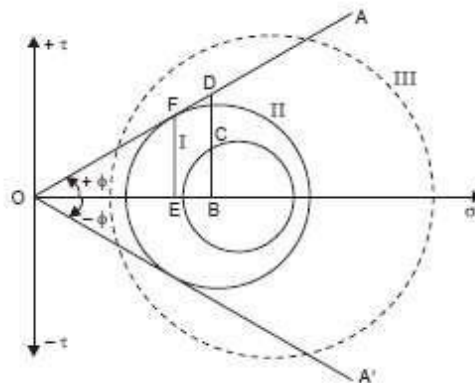


Fig.11.4: Mohr's strength theory—Mohr envelopes for cohesionless soil

If the stress conditions at a point are represented by Mohr's circle I, the shear stress on any plane through the point is less than the shearing strength, as indicated by the line  $BCD$ ;

$BC$  represents the shear stress on a plane on which the normal stress is given by  $OD$ .  $BD$ , representing the shearing strength for this normal stress, is greater than  $BC$ .

The stress conditions represented by the Mohr's Circle II, which is tangential to the Mohr's envelope at  $F$ , are such that the shearing stress,  $EF$ , on the plane of maximum obliquity is equal to the shearing strength. Failure is incipient on this plane and will occur unless the normal stress on the critical plane increases.

It may be noted that it would be impossible to apply the stress conditions represented by Mohr's circle III (dashed) to this soil sample, since failure would have occurred even by the time the shear stress on the critical plane equals the shearing strength available on that plane, thus eliminating the possibility of the shear stress exceeding the shearing strength.

The Mohr's strength theory, or theory of failure or rupture, may thus be stated as follows:

The stress condition given by any Mohr's circle falling within the Mohr's envelope represents a condition of stability, while the condition given by any Mohr's circle tangent to the Mohr's envelope indicates incipient failure on the plane relating to the point of tangency. The Mohr's envelope may be treated to be a property of the material and independent of the imposed stresses. Also, the Mohr's circle of stress depends only upon the imposed stresses and has nothing to do with the nature and properties of the material.

To emphasize that the stresses in Eq. 11.18 are those on the plane on which failure is incipient, we add the subscript  $f$  to  $\sigma$ :

$$s = \sigma_f \tan \phi \text{-----} \text{Eq.11.19}$$

It is possible to express the strength in terms of normal stress on any plane, with the aid of the Mohr's circle of stress. Some common relationships are :

$$\sigma_f = \sigma_3(1 + \sin \phi) = \sigma_1(1 - \sin \phi) \text{-----} \text{Eq.11.20}$$

$$s = \sigma_f \tan \phi = \frac{\sigma_1 - \sigma_3}{2} \cos \phi \text{-----} \text{Eq.11.21}$$

The primary assumptions in the Mohr's strength theory are that the intermediate principal stress has no influence on the strength and that the strength is dependent only upon the normal stress on the plane of maximum obliquity. However, the shearing strength, in fact, does depend to a small extent upon the intermediate principal stress, density speed of application of shear, and so on. But the Mohr theory explains satisfactorily the strength concept in soils and hence is in vogue. It may also be noted that the Mohr envelope will not be a straight line but is actually slightly curved since the angle of internal friction is known to decrease slightly with increase in stress.

### 11.2.3 Mohr-Coulomb Theory

The Mohr-Coulomb theory of shearing strength of a soil, first propounded by Coulomb (1776) and later generalised by Mohr, is the most commonly used concept. The functional relationship between the normal stress on any plane and the shearing strength available on that plane was assumed to be linear by Coulomb; thus the following is usually known as Coulomb's law:

$$s = c + \sigma \tan \phi \text{ -----}$$

Eq. 11.22

where  $c$  and  $\phi$  are empirical parameters, known as the 'apparent cohesion' and 'angle of shearing resistance' (or angle of internal friction), respectively. These are better visualised as 'parameters' and not as absolute properties of a soil since they are known to vary with water content, conditions of testing such as speed of shear and drainage conditions, and a number of other factors besides the type of soil.

Coulomb's law is merely a mathematical equation of the failure envelope shown in Figure 11.5 (a); Mohr's generalisation of the failure envelope as a curve which becomes flatter with increasing normal stress is shown in Fig. 11.5 (b).

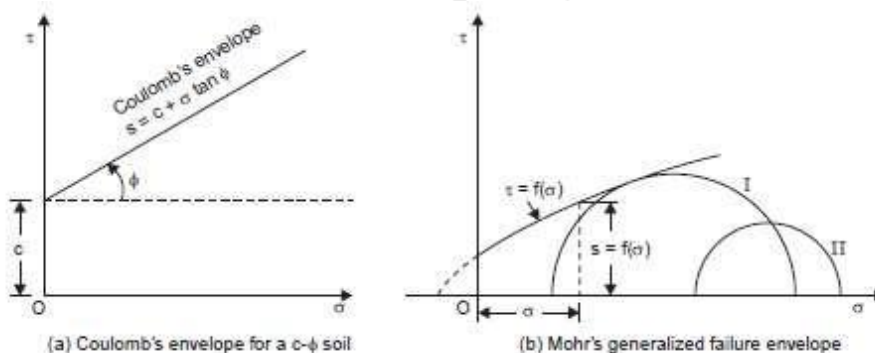


Fig. 11.5: Mohr-Coulomb Theory—failure envelopes

The envelopes are called 'strength envelopes' or 'failure envelopes'. The meaning of an envelope has already been given in the previous section; if the normal and shear stress components on a plane are plotted on to the failure envelope, failure is supposed to be incipient and if the stresses plot below the envelope, the condition represents stability. And, it is impossible that these plot above the envelope, since failure should have occurred previously. Coulomb's law is also written as follows to indicate that the stress condition refers to that on the plane of failure:

$$s = c + \sigma_f \tan \phi \text{ -----}$$

Eq. 11.23

In a different way, it can be said that the Mohr's circle of stress relating to a given stress condition would represent, incipient failure condition if it just touches or is tangent to the strength or failure envelope (circle I); otherwise, it would wholly lie below the envelopes as shown in circle II, Fig.11.5 (b).

The Coulomb envelope in special cases may take the shapes given in Fig. 11.6 (a) and (b); for a purely cohesionless or granular soil or a pure sand, it would be as shown in Fig. 11.6 (a) and for a purely cohesive soil or a pure clay, it would be as shown in Fig. 11.6 (b).

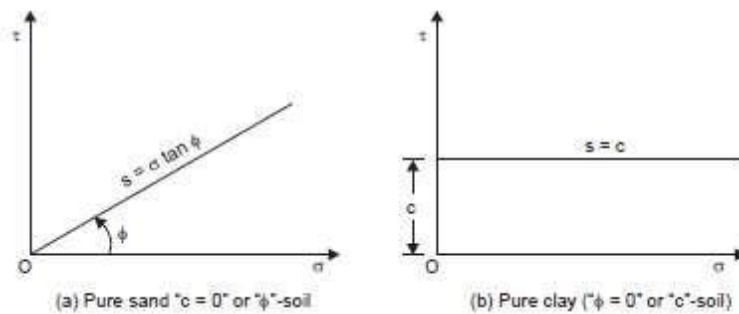


Fig. 11.6: Coulomb envelopes for pure sand and for pure clay

### 11.3 SHEARING STRENGTH—A FUNCTION OF EFFECTIVE STRESS

Equation 11.23 apparently indicates that the shearing strength of a soil is governed by the *total* normal stress on the failure plane. However, according to Terzaghi, it is the effective stress on the failure plane that governs the shearing strength and not the total stress.

It may be expected intuitively that the denser a soil, the greater the shearing strength. It has been learnt in chapter seven that a soil deposit becomes densest under any given pressure after the occurrence of complete consolidation and consequent dissipation of pore water

Thus, complete consolidation, dependent upon the dissipation of pore water pressure and hence upon the increase in the effective stress, leads to increase in the shearing strength of a soil. In other words, it is the effective stress in the case of a saturated soil and not the total stress which is relevant to the mobilisation of shearing stress.

Further, the density of a soil increase when subjected to shearing action, drainage being allowed simultaneously. Therefore, even if two soils are equally dense on having been consolidated to the same effective stress, they will exhibit different shearing strengths if drainage is permitted during shear for one, while it is not for the other.

These ideas lead to a statement that “the strength of a soil is a unique function of the effective stress acting on the failure plane”.

Eq. 11.23 may now be modified to read:

$$s = c' + \sigma_f \tan \phi' \text{ -----}$$

Eq. 11.24

Where,  $c'$  and  $\phi'$  are called the effective cohesion and effective angle of internal friction, respectively, since they are based on the effective normal stress on the failure plane.



Collectively, they are called ‘effective stress parameters’, while  $c$  and  $\phi$  of Eq. 24 are called ‘‘total stress parameters’’.

## 11.4 SHEARING STRENGTH TESTS

Determination of shearing strength of a soil involves the plotting of failure envelopes and evaluation of the shear strength parameters for the necessary conditions. The following tests are available for this purpose:

### ***Laboratory Tests***

1. Direct Shear Test
2. Triaxial Compression Test
3. Unconfined Compression Test
4. Laboratory Vane Shear Test
5. Torsion Test
6. Ring Shear Tests

### ***Field Tests***

1. Vane Shear Test
2. Penetration Test

The first three tests among the laboratory tests are very commonly used, while the fourth is gaining popularity owing to its simplicity. The fifth and sixth are mostly used for research purposes and hence are not dealt with here.

The principle of the field vane test is the same as that of the laboratory vane shear test, except that the apparatus is bigger in size for convenience of field use. The penetration test involves the measurement of resistance of a soil to penetration of a cone or a cylinder, as an indication of the shearing strength. This procedure is indirect and rather empirical in nature although correlations are possible. The field tests are also not considered here. The details of the test procedures are available in the relevant I.S. codes or any book on laboratory testing, such as Lambe (1951).

### ***11.4.1 Direct Shear Test***

The direct shear device, also called the ‘shear box apparatus’, essentially consists of a brass box, split horizontally at mid-height of the soil specimen, as shown schematically in Figure 11.7.

The soil is gripped in perforated metal grilles, behind which porous discs can be placed if required to allow the specimen to drain. For undrained tests, metal plates and solid metal grilles may be used. The usual plan size of the specimen is 60 mm square; but a larger size such as 300 mm square or even more, is employed for testing larger size granular material such as gravel. The minimum thickness or height of the specimen is 20 mm. After the sample to be tested is placed in the apparatus or shear box, a normal load which is vertical is applied to the top of the sample by means of a loading yoke and weights. Since the shear plane is predetermined as the horizontal plane, this becomes the normal stress on the failure plane, which is kept constant throughout the test. A shearing force is applied to the upper-half of the box, which is zero initially and is increased until the specimen fails.

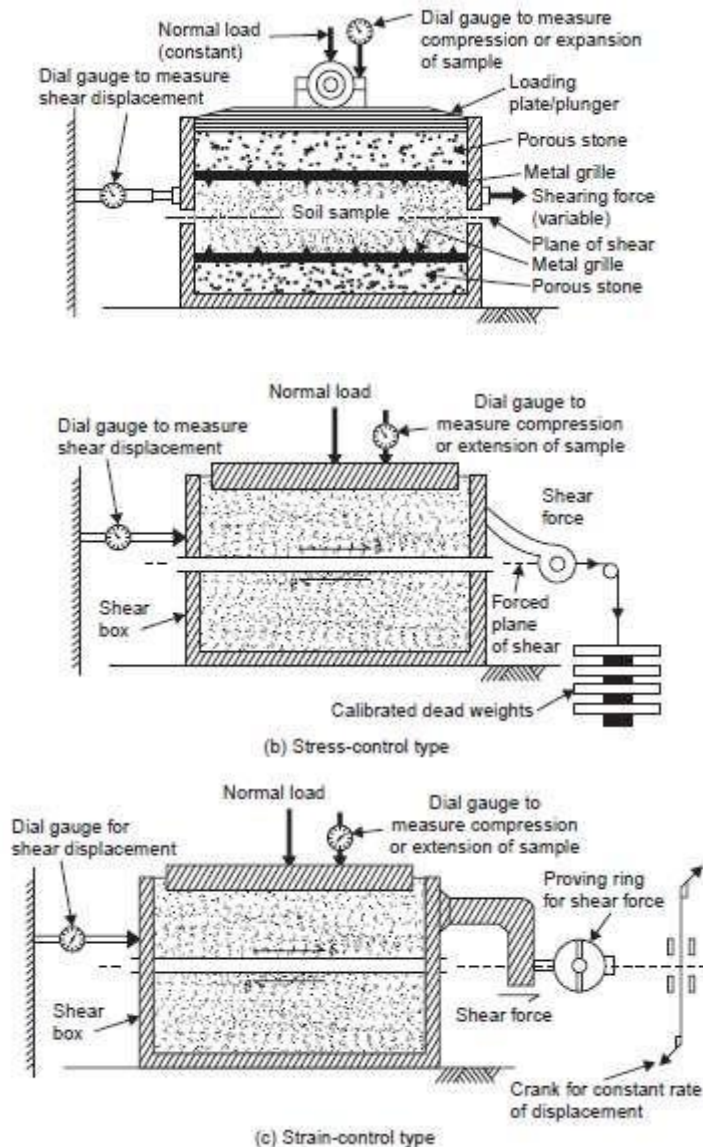


Fig. 11.7: Direct Shear Test Device

Two types of application of shear are possible—one in which the shear stress is controlled and the other in which the shear strain is controlled. The principles of these two types of devices are illustrated schematically in Fig. 11.7 (b) and (c), respectively. In the stress-controlled type, the shear stress, which is the controlled variable, may be applied at a constant rate or more commonly in equal increments by means of calibrated weights hung from a hanger attached to a wire passing over a pulley. Each increment of shearing force is applied and held constant, until the shearing deformation ceases. The shear displacement is measured with the aid of a dial gauge attached to the side of the box. In the strain-controlled type, the shear displacement is applied at a constant rate by means of a screw operated manually or by motor. With this type of test the shearing force necessary to overcome the resistance within the soil is automatically developed. This shearing force is measured with the aid of a proving ring—a steel ring that has been

carefully machined, balanced and calibrated. The deflection of the annular ring is measured with the aid of a dial gauge set inside the ring, the causative force being got for any displacement by means of the calibration chart supplied by the manufacturer.

The shear displacement is measured again with the aid of another dial gauge attached to the side of the box

In both cases, a dial gauge attached to the plunger, through which the normal load is applied, will enable one to determine the changes in the thickness of the soil sample which will help in the computation of volume changes of the sample, if any. The strain-controlled type is very widely used. The strain is taken as the ratio of the shear displacement to the thickness of the sample. The proving ring readings may be taken at fixed displacements or even at fixed intervals of time as the rate of strain is made constant by an electric motor. A sudden drop in the proving ring reading or a leveling-off in successive readings indicates shear failure of the soil specimen.

The shear strain may be plotted against the shear stress; it may be plotted versus the ratio of the shearing stress on normal stress; and it may also be plotted versus volume change.

Each plot may yield information useful in one way or the other. The stresses may be obtained from the forces by dividing them by the area of cross-section of the sample. The stress- conditions on the failure plane and the corresponding Mohr's circle for direct shear test are shown in Fig. 11.8 (a) and (b) respectively.

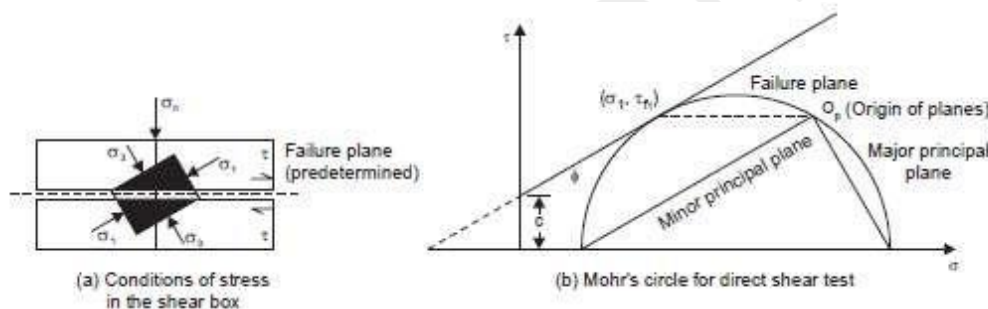


Fig.11.8: Mohr's circle representation of stress condition for direct shear test results

The failure plane is predetermined as the horizontal plane here. Several specimens are tested under different normal loads and the results plotted to obtain failure envelopes.

The direct shear test is a relatively simple test. Quick drainage, *i.e.*, quick dissipation of pore pressures is possible since the thickness of the specimen is small. However, the test suffers from the following inherent **disadvantages**, which limit its application.

1. The stress conditions are complex primarily because of the non-uniform distribution of normal and shear stresses on the plane.
2. There is virtually no control of the drainage of the soil specimen as the water content of a saturated soil changes rapidly with stress.
3. The area of the sliding surface at failure will be less than the original area of the soil specimen and strictly speaking, this should be accounted for.
4. The ridges of the metal gratings embedded on the top and bottom of the specimen, causes distortion of the specimen to some degree.

5. The effect of lateral restraint by the side walls of the shear box is likely to affect the results.
6. The failure plane is predetermined and this may not be the weakest plane. In fact, this is the most important limitation of the direct shear test.

#### 11.4.2 Triaxial Shear Test

The triaxial shear test is one of the most reliable methods available for determining shear strength parameters. It is used widely for research and conventional testing. A diagram of the triaxial test layout is shown in Figure 11.9.

In this test, a soil specimen about 36 mm in diameter and 76 mm (3 in.) long generally is used. The specimen is encased by a thin rubber membrane and placed inside a plastic cylindrical chamber that usually is filled with water or glycerine. The specimen is subjected to a confining pressure by compression of the fluid in the chamber. (*Note:* Air is sometimes used as a compression medium.) To cause shear failure in the specimen, one must apply axial stress (sometimes called *deviator stress*) through a vertical loading ram.

This stress can be applied in one of two ways:

1. Application of dead weights or hydraulic pressure in equal increments until the specimen fails. (Axial deformation of the specimen resulting from the load applied through the ram is measured by a dial gauge.)
2. Application of axial deformation at a constant rate by means of a geared or hydraulic loading press. This is a strain-controlled test.

The axial load applied by the loading ram corresponding to a given axial deformation is measured by a proving ring or load cell attached to the ram.

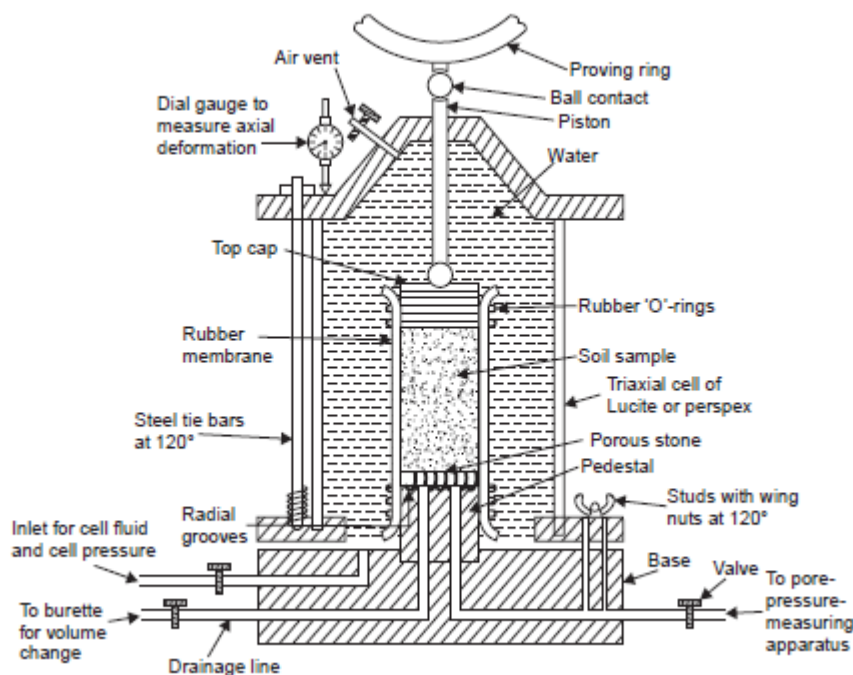


Fig.11.9: Triaxial experimental set up

**Test Procedure**

The essential steps in the conduct of the test are as follows:

- (i) A saturated porous stone is placed on the pedestal and the cylindrical soil specimen is placed on it.
- (ii) The specimen is enveloped by a rubber membrane to isolate it from the water with which the cell is to be filled later; it is sealed with the pedestal and top cap by rubber “O” rings.
- (iii) The cell is filled with water and pressure is applied to the water, which in turn is transmitted to the soil specimen all-round and at top. This pressure is called ‘cell pressure’, ‘chamber pressure’ or ‘confining pressure’.
- (iv) Additional axial stress is applied while keeping the cell pressure constant. This introduces shearing stresses on all planes except the horizontal and vertical planes, on which the major, minor and intermediate principal stresses act, the last two being equal to the cell pressure on account of axial symmetry.
- (v) The additional axial stress is continuously increased until failure of the specimen occurs. (What constitutes failure is often a question of definition and may be different for different kinds of soils. This aspect would be discussed later on).

A number of observations may be made during a Triaxial compression test regarding the physical changes occurring in the soil specimen:

- (a) As the cell pressure is applied, pore water pressure develops in the specimen, which can be measured with the help of a pore pressure measuring apparatus, such as Bishop’s pore pressure device (Bishop, 1960), connected to the pore pressure line, after closing the valve of the drainage line.
- (b) If the pore pressure is to be dissipated, the pore water line is closed, the drainage line opened and connected to a burette. The volume decrease of the specimen due to consolidation is indicated by the water drained into the burette.
- (c) The axial strain associated with the application of additional axial stress can be measured by means of a dial gauge, set to record the downward movement of the loading piston.
- (d) Upon application of the additional axial stress, some pore pressure develops. It may be measured with the pore pressure device, after the drainage line is closed. On the other hand, if it is desired that any pore pressure developed be allowed to be dissipated, the pore water line is closed and the drainage line opened as stated previously.
- (e) The cell pressure is measured and kept constant during the course of the test.
- (f) The additional axial stress applied is also measured with the aid of a proving ring and dial gauge.

Thus the entire triaxial test may be visualised in two important stages:

- (i) The specimen is placed in the triaxial cell and cell pressure is applied during the first stage.
- (ii) The additional axial stress is applied and is continuously increased to cause a shear failure, the potential failure plane being that with maximum obliquity during the second stage.

### 11.4.3 Area Correction for the Determination of Additional Axial Stress or Deviatoric Stress

The additional axial load applied at any stage of the test can be determined from the proving ring reading. During the application of the load, the specimen undergoes axial compression and horizontal expansion to some extent. Little error is expected to creep in if the volume is supposed to remain constant, although the area of cross-section varies as axial strain increases.

The assumption is perfectly valid if the test is conducted under undrained conditions, but, for drained conditions, the exact relationship is somewhat different.

If  $A_0$ ,  $h_0$  and  $V_0$  are the initial area of cross-section, height and volume of the soil specimen respectively, and if  $A$ ,  $h$ , and  $V$  are the corresponding values at any stage of the test, the corresponding changes in the values being design

$$A(h_0 - \Delta h) = V = V_0 + \Delta V \text{-----} \quad \text{Eq.11.25}$$

$$\text{Hence } A = \frac{V_0 + \Delta V}{h_0 - \Delta h} \text{-----} \quad \text{Eq.11.26}$$

But, for axial compression,  $\Delta h$  is known to be negative.

$$A = \frac{V_0 + \Delta V}{h_0 - \Delta h} = \frac{V_0(1 + \frac{\Delta V}{V_0})}{h_0(1 - \frac{\Delta h}{h_0})} = \frac{A_0(1 + \frac{\Delta V}{V_0})}{1 - \epsilon_a} \text{-----} \quad \text{Eq.11.27}$$

$$\text{since the axial strain, } \epsilon_a = \frac{\Delta h}{h_0} \text{-----} \quad \text{Eq.11.28}$$

$$\text{For an undrained test, } A = \frac{A_0}{1 - \epsilon_a} \text{-----} \quad \text{Eq.11.29}$$

since  $\Delta V = 0$ .

This is called the 'Area correction' and  $\frac{1}{1 - \epsilon_a}$  is the correction factor.

A more accurate expression for the corrected area is given by

$$A = \frac{A_0}{1 - \epsilon_a} = \frac{V_0 + \Delta V}{h_0 - \Delta h} \text{-----} \quad \text{Eq.11.30}$$

Once the corrected area is determined, the additional axial stress or the deviator stress,  $\Delta\sigma$ , is obtained as

$$\Delta\sigma = \sigma_1 - \sigma_3 = \text{Axial load (from proving ring reading) / Corrected area}$$

The cell pressure or the confining pressure,  $\sigma_c$ , itself being the minor principal stress,  $\sigma_3$ , this is constant for one test; however, the major principal stress,  $\sigma_1$ , goes on increasing until failure.

$$\sigma_1 = \sigma_3 + \Delta\sigma \text{-----} \quad \text{Eq.11.31}$$

### 11.4.4 Mohr's Circle for Triaxial Test

The stress conditions in a triaxial test may be represented by a Mohr's circle, at any stage of the

test, as well as at failure, as shown in Figure 11.10:

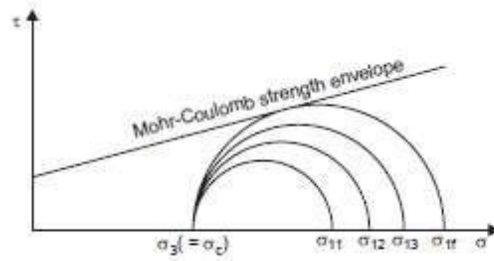


Fig. 11.10: Mohr's circle during Triaxial test

The cell pressure,  $\sigma_c$  which is also the minor principal stress is constant and  $\sigma_{11}$ ,  $\sigma_{12}$ ,  $\sigma_{13}$ ,  $\sigma_{1f}$  are the major principal stresses at different stages of loading and at failure. The Mohr's circle at failure will be tangential to the Mohr-Coulomb strength envelope, while those at intermediate stages will be lying wholly below it. The Mohr's circle at failure for one particular value of cell pressure will be as shown in Fig. 11.11.

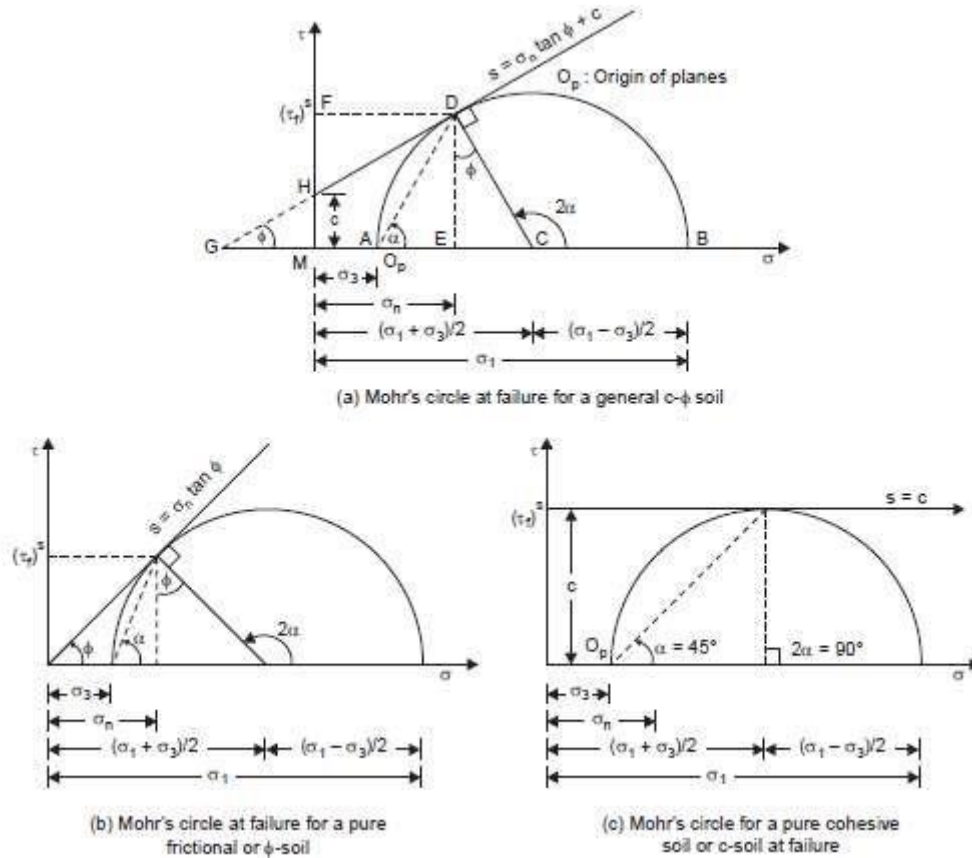


Fig. 11.11: Mohr's circle at failure for a particular cell pressure

The Mohr's circles at failure for one particular cell pressure are shown for the three typical cases of a general  $c$ - $\phi$  soil, a  $\phi$ -soil and a  $c$ -soil in Figs. 11.11 (a), (b), and (c) respectively.

With reference to Fig. 11.11 (a), the relationship between the major and minor principal stresses at failure may be established from the geometry of the Mohr's circle, as follows:

From  $\triangle DCG$ ,  $2\alpha = 90^\circ + \phi$

$$\therefore \alpha = 45^\circ + \phi/2 \text{-----}$$

Eq.11.32

Again from  $\triangle DCG$

$$\sin \phi = \frac{DC}{GC} = \frac{DC}{GM+MC} = \frac{\frac{\sigma_1 - \sigma_3}{2}}{ccot\phi + \frac{\sigma_1 + \sigma_3}{2}} \text{-----}$$

Eq.11.33

$$\sigma_1 - \sigma_3 = 2ccot\phi + (\sigma_1 + \sigma_3)\sin\phi \text{-----}$$

Eq.11.34

$$\text{Or, } \sigma_1 = \sigma_3 \tan^2 (45^\circ + \phi/2) + 2c \tan(45^\circ + \phi/2) \text{-----}$$

Eq.11.35



$$\text{Or , } \sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha \text{ -----}$$

Eq.11.36

This is also written as

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi} \text{-----} \text{Eq.11.37}$$

$$\text{where, } N_\phi = \tan^2 \alpha = \tan^2(45^\circ + \frac{\phi}{2}) \text{-----} \text{Eq.11.38}$$

The above equations define the relationship between the principal stresses at failure. This state of stress is defined as ‘Plastic equilibrium condition’, when failure is imminent.

From one test, a set of  $\sigma_1$  and  $\sigma_3$  is known; however, it can be seen from that at least two such sets are necessary to evaluate the parameters  $c$  and  $\phi$  conventionally, three or more such sets are used from a corresponding number of tests.

The usual procedure is to plot the Mohr’s circles for a number of tests and take the best common tangent to the circles as the strength envelope. A small curvature occurs in the strength envelope of most soils, but since this effect is slight, the envelope for all practical purposes, may be taken as a straight line. The intercept of the strength envelope on the  $\tau$ -axis gives the cohesion and the angle of slope of this line with  $\sigma$ -axis gives the angle of internal friction, as shown in Fig. 11.12.

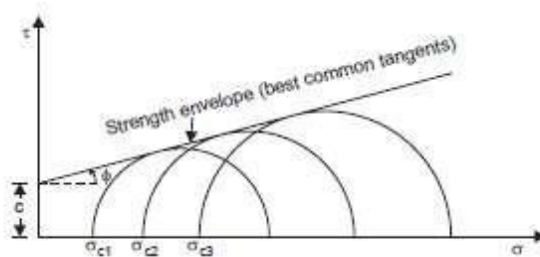


Fig.11.12: Mohr’s circle at failure for different cell pressures

#### 11.4.5 Merits of Triaxial Compression Test

The following are the significant points of merit of triaxial compression test:

- (1) Failure occurs along the weakest plane unlike along the predetermined plane in the case of direct shear test.
- (2) The stress distribution on the failure plane is much more uniform than it is in the direct shear test: the failure is not also progressive, but the shear strength is mobilised all at once. Of course, the effect of end restraint for the sample is considered to be a disadvantage; however, this may not have pronounced effect on the results since the conditions are more uniform to the desired degree near the middle of the height of the sample where failure usually occurs.
- (3) Complete control of the drainage conditions is possible with the triaxial compression test; this would enable one to simulate the field conditions better.
- (4) The possibility to vary the cell pressure or confining pressure also affords another means to simulate the field conditions for the sample, so that the results are more meaningfully interpreted.
- (5) Precise measurements of pore water pressure and volume changes during the test are possible.
- (6) The state of stress within the specimen is known on all planes and not only on a predetermined failure plane as it is with direct shear tests.
- (7) The state of stress on any plane is capable of being determined not only at failure but also at any earlier stage.

- (8) Special tests such as extension tests are also possible to be conducted with the triaxial testing apparatus.
- (9) It provides an ingenious and a symmetrical three-dimensional stress system better suited to simulate field conditions.

### 11.5 TYPES OF SHEAR TESTS BASED ON DRAINAGE CONDITIONS

Before considering various methods of conducting shearing strength tests on a soil, it is necessary to consider the possible drainage conditions before and during the tests since the results is significantly affected by these.

A cohesionless or a coarse-grained soil may be tested for shearing strength either in the dry condition or in the saturated condition. A cohesive or fine-grained soil is usually tested in the saturated condition. Depending upon whether drainage is permitted before and during the test, shear tests on such saturated soils are classified as follows:

#### ***Unconsolidated Undrained Test***

Drainage is not permitted at any stage of the test, that is, either before the test during the application of the normal stress or during the test when the shear stress is applied. Hence no time is allowed for dissipation of pore water pressure and consequent consolidation of the soil; also, no significant volume changes are expected. Usually, 5 to 10 minutes may be adequate for the whole test, because of the shortness of drainage path. However, undrained tests are often performed only on soils of low permeability. This is the most unfavourable condition which might occur in geotechnical engineering practice and hence is simulated in shear testing. Since a relatively small time is allowed for the testing till failure, it is also called the 'Quick test.' It is designated *UU*, *Q*, or *Qu* test.

#### ***Consolidated Undrained Test***

Drainage is permitted fully in this type of test during the application of the normal stress and no drainage is permitted during the application of the shear stress. Thus volume changes do not take place during shear and excess pore pressure develops. Usually, after the soil is consolidated under the applied normal stress to the desired degree, 5 to 10 minutes may be adequate for the test. This test is also called 'consolidated quick test' and is designated *CU* or *Qc* test, These conditions are also common in geotechnical engineering practice.

#### ***11.5.1 Consolidated-Drained Triaxial Test***

In the CD test, the saturated specimen first is subjected to an all around confining pressure,  $\sigma_3$ , by compression of the chamber fluid. As confining pressure is applied, the pore water pressure of the specimen increases by  $u_c$  (if drainage is prevented). This increase in the pore water pressure can be expressed as a non dimensional parameter in the form

$$B = \frac{u_c}{\sigma_3} \text{-----} \quad \text{Eq.11.39}$$

Where, B= Skempton's pore pressure parameter (Skempton, 1954).

For saturated soft soils,  $B$  is approximately equal to 1; however, for saturated stiff soils, the magnitude of  $B$  can be less than 1. Black and Lee (1973) gave the theoretical values of  $B$  for various soils at complete saturation.

Now, if the connection to drainage is opened, dissipation of the excess pore water pressure, and thus consolidation, will occur. With time,  $u_e$  will become equal to 0. In saturated soil, the change in the volume of the specimen ( $\Delta V_c$ ) that takes place during consolidation can be obtained from the volume of pore water drained (Figure 11.13a).

Next, the deviator stress,  $\Delta\sigma_d$  on the specimen is increased very slowly (Figure 11.13b). The drainage connection is kept open, and the slow rate of deviator stress application allows complete dissipation of any pore water pressure that developed as a result ( $\Delta u_d=0$ )

A typical plot of the variation of deviator stress against strain in loose sand and normally consolidated clay is shown in Figure 11.13b. Figure 1c shows a similar plot for dense sand and over consolidated clay. The volume change,  $\Delta V_d$ , of specimens that occurs because of the application of deviator stress in various soils is also shown in Figures 11.13d and e.

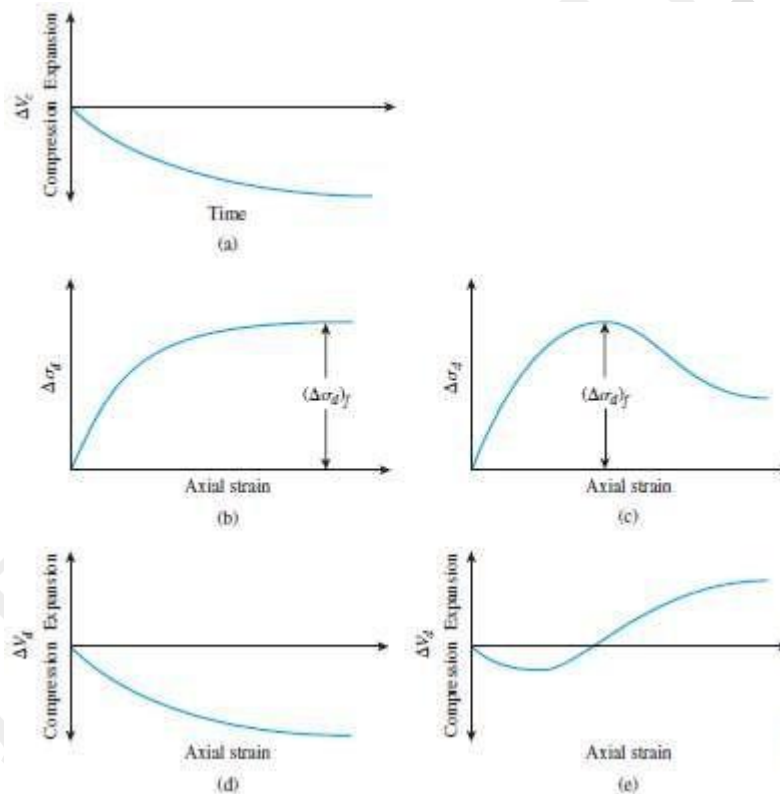


Fig. 11.13: Consolidated-drained triaxial test: (a) volume change of specimen caused by chamber-confining pressure; (b) plot of deviator stress against strain in the vertical direction for loose sand and normally consolidated clay; (c) plot of deviator stress against strain in the vertical direction for dense sand and over consolidated clay; (d) volume change in loose sand and normally consolidated clay during deviator stress application; (e) volume change in dense sand and over consolidated clay during deviator stress application

Because the pore water pressure developed during the test is completely dissipated, we have

Total and effective confining stress =  $\sigma_3 = \sigma'_3$

Total and effective axial stress at failure =  $\sigma_3 + \Delta\sigma_{df} = \sigma_1 = \sigma'_1$

In a triaxial test,  $\sigma'_1$  is the major principal effective stress at failure and is the minor principal effective stress at failure. Several tests on similar specimens can be conducted by varying the confining pressure. With the major and minor principal stresses at failure for each test the Mohr's circles can be drawn and the failure envelopes can be obtained. Figure 11.14 shows the type of effective stress failure envelope obtained for tests on sand and normally consolidated clay.

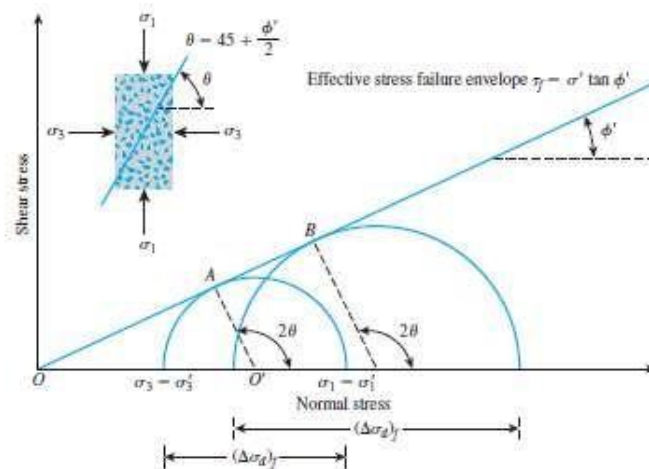


Fig.11.14: Effective stress failure envelope from drained tests on sand and normally consolidated clay

The coordinates of the point of tangency of the failure envelope with a Mohr's circle (that is, point A) give the stresses (normal and shear) on the failure plane of that test specimen.

For normally consolidated clay, referring to Figure 2

$$\sin\phi' = \frac{OA}{OO'} \quad \text{Eq.11.40}$$

$$\text{OR } \sin\phi' = \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3} \quad \text{Eq.11.41}$$

$$\text{Hence } \phi' = \sin^{-1} \left( \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3} \right) \quad \text{Eq.11.42}$$

Also, the failure plane will be inclined at an angle of  $\theta = 45 + \frac{\phi'}{2}$  to the major principal plane, as shown in Figure 11.14.

Over consolidation results when a clay initially is consolidated under an all-around chamber pressure of  $\sigma_c = \sigma'_c$  and is allowed to swell by reducing the chamber pressure to  $\sigma_3 = \sigma'_3$ . The

failure envelope obtained from drained triaxial tests of such over consolidated clay specimens shows two distinct branches (*ab* and *bc* in Figure 11.15). The portion *ab* has a flatter slope with a cohesion intercept, and the shear strength equation for this branch can be written as

$$\tau_f = c' + \sigma' \tan\phi' \quad \text{Eq.11.43}$$

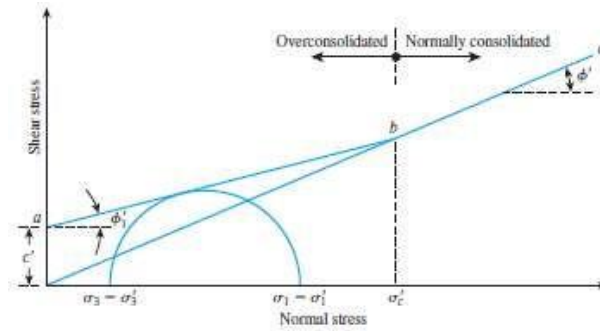


Fig. 11.15: Effective stress failure envelope for over consolidated clay

A consolidated-drained triaxial test on a clayey soil may take several days to complete. This amount of time is required because deviator stress must be applied very slowly to ensure full drainage from the soil specimen. For this reason, the CD type of triaxial test is uncommon.

### 11.5.2 Consolidated-Undrained Triaxial Test

The consolidated-undrained test is the most common type of triaxial test. In this test, the saturated soil specimen is first consolidated by an all-around chamber fluid pressure,  $\sigma_3$ , that results in drainage (Figures 11.16 a and b).

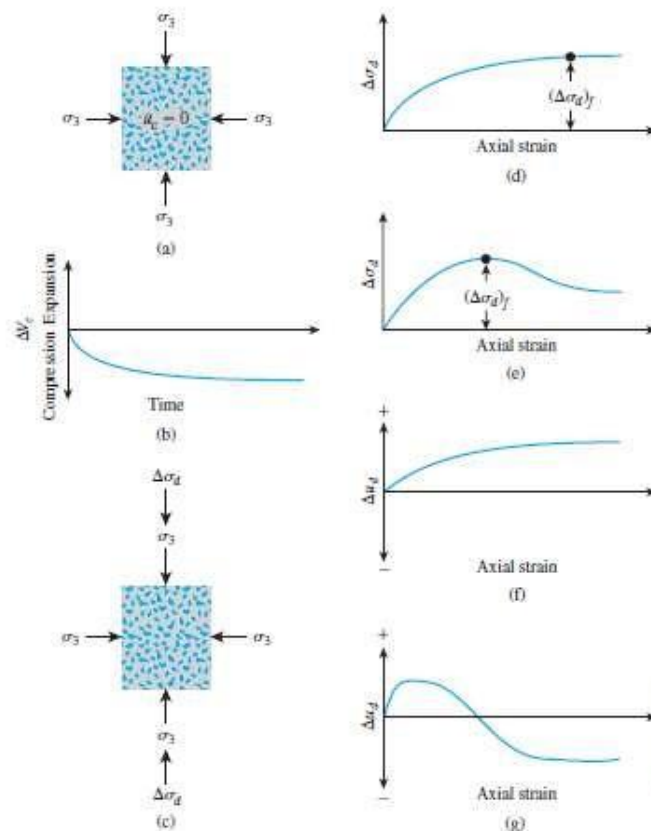


Fig.11.16: Consolidated-undrained test: (a) specimen under chamber-confining pressure; (b) volume change in specimen caused by confining pressure; (c) deviator stress application; (d)

deviator stress against axial strain for loose sand and normally consolidated clay; (e) deviator stress against axial strain for dense sand and over consolidated clay; (f) variation of pore water pressure with axial strain for loose sand and normally consolidated clay; (g) variation of pore water pressure with axial strain for dense sand and over consolidated clay.

After the pore water pressure generated by the application of confining pressure is dissipated, the deviator stress,  $\Delta\sigma_d$ , on the specimen is increased to cause shear failure (Figure 11.16c). During this phase of the test, the drainage line from the specimen is kept closed. Because drainage is not permitted, the pore water pressure,  $\Delta u_d$ , will increase. During the test, simultaneous measurements of  $\Delta\sigma_d$ , and  $\Delta u_d$  are made. The increase in the pore water pressure,  $\Delta u_d$  can be expressed in a nondimensional form as

$$\bar{A} = \frac{\Delta u_d}{\Delta\sigma_d} \text{-----} \quad \text{Eq.11.44}$$

where  $\bar{A}$  = Skempton's pore pressure parameter (Skempton, 1954).

The general patterns of variation of  $\Delta\sigma_d$  and  $\Delta u_d$  with axial strain for sand and clay soils are shown in Figures 4 d through 4g. In loose sand and normally consolidated clay, the pore water pressure increases with strain. In dense sand and over consolidated clay, the pore water pressure increases with strain to a certain limit, beyond which it decreases and becomes negative (with respect to the atmospheric pressure). This decrease is because of a tendency of the soil to dilate. Unlike the consolidated-drained test, the total and effective principal stresses are not the same in the consolidated-undrained test. Because the pore water pressure at failure is measured in this test, the principal stresses may be analyzed as follows:

- Major principal stress at failure (total):
- Major principal stress at failure (effective):
- Minor principal stress at failure (total):
- Minor principal stress at failure (effective):

The preceding derivations show that

$$\sigma_1 - \sigma_3 = \sigma'_1 - \sigma'_3 \text{-----} \quad \text{Eq.11.45}$$

Tests on several similar specimens with varying confining pressures may be conducted to determine the shear strength parameters. Figure 11.17 shows the total and effective stress

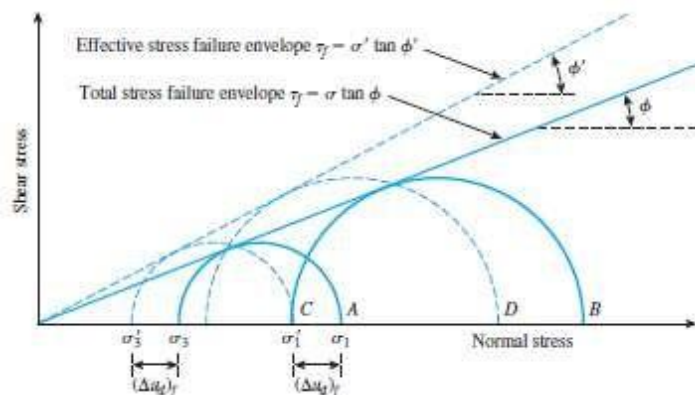


Fig. 11.17: Total and effective stress failure envelopes for consolidated undrained triaxial tests

Mohr's circles at failure obtained from consolidated-undrained triaxial tests in sand and normally consolidated clay. Note that  $A$  and  $B$  are two total stress Mohr's circles obtained from two tests.  $C$  and  $D$  are the effective stress Mohr's circles corresponding to total stress circles  $A$  and  $B$ , respectively. The diameters of circles  $A$  and  $C$  are the same; similarly, the diameters of circles  $B$  and  $D$  are the same. In Figure 11.17, the total stress failure envelope can be obtained by drawing a line that touches all the total stress Mohr's circles. For sand and normally consolidated clays, this will be approximately a straight line passing through the origin and may be expressed by the equation

$$\tau_f = \sigma \tan \phi \text{-----} \quad \text{Eq.11.46}$$

where  $\sigma$  = total stress

$\phi$  = the angle that the total stress failure envelope makes with the normal stress axis, also known as the *consolidated-undrained angle of shearing resistance*

For sand and normally consolidated clay, we can write

$$\phi = \sin^{-1} \left( \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \right) \text{-----} \quad \text{Eq.11.47}$$

And

$$\phi' = \sin^{-1} \left( \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2(\Delta u_d)} \right) \text{-----} \quad \text{Eq.11.48}$$

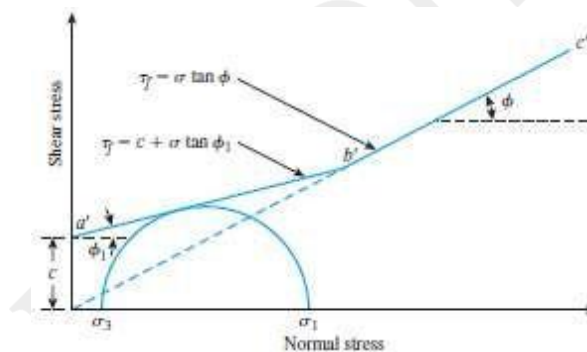


Fig. 11.18: Total stress failure envelope obtained from consolidated-undrained tests in over consolidated clay

Again referring to Fig.11.17, we see that the failure envelope that is tangent to all the effective stress Mohr's circles can be represented by the equation

$$\tau_f = \sigma' \tan \phi' \text{-----} \quad \text{Eq.11.48}$$

which is the same as that obtained from consolidated-drained tests.

In over consolidated clays, the total stress failure envelope obtained from consolidated undrained tests will take the shape shown in Fig.11.18. The straight line is represented by the equation

$$\tau_f = \sigma \tan \phi_1 \text{-----} \quad \text{Eq.11.49}$$

and the straight line follows the relationship given by the above equation. The effective stress failure envelope drawn from the effective stress Mohr's circles will be similar to that shown in Figure 11.15.

Consolidated-drained tests on clay soils take considerable time. For this reason, consolidated-undrained tests can be conducted on such soils with pore pressure measurements to obtain the drained shear strength parameters. Because drainage is not allowed in these tests during the



application of deviator stress, they can be performed quickly. Skempton's pore water pressure parameter was defined as follows. At failure, the parameter can be written as

$$A = \bar{A} = \frac{\Delta u_{df}}{\Delta \sigma_{df}} \quad \text{Eq.11.50}$$

### 11.5.3 Unconsolidated-Undrained Triaxial Test

In unconsolidated-undrained tests, drainage from the soil specimen is not permitted during the application of chamber pressure  $\sigma_3$ . The test specimen is sheared to failure by the application of deviator stress,  $\Delta \sigma_d$ , and drainage is prevented. Because drainage is not allowed at any stage, the test can be performed quickly. Because of the application of chamber confining pressure,  $\sigma_3$ , the pore water pressure in the soil specimen will increase by  $u_c$ . A further increase in the pore water pressure ( $\Delta u_d$ ) will occur because of the deviator stress application. Hence, the total pore water pressure  $u$  in the specimen at any stage of deviator stress application can be given as

$$u = u_c + \Delta u_d \quad \text{Eq.11.51}$$

But we know that  $u_c = B\sigma_3$  and  $\Delta u_c = A\Delta \sigma_d$

Now substituting above relation, we obtain

$$u = B\sigma_3 + A\Delta \sigma_d \quad \text{Eq.11.52}$$

This test usually is conducted on clay specimens and depends on a very important strength concept for cohesive soils if the soil is fully saturated. The added axial stress at failure  $\Delta \sigma_{df}$  is practically the same regardless of the chamber confining pressure. This property is shown in Figure 7. The failure envelope for the total stress Mohr's circles becomes a horizontal line and hence is called a  $\phi = 0$  condition. So we get

$$\tau_f = c = c_u \quad \text{Eq.11.53}$$

Where,  $c_u$  is the undrained shear strength and is equal to the radius of the Mohr's circles.

Note that the  $\phi = 0$  concept is applicable to only saturated clays and silts.

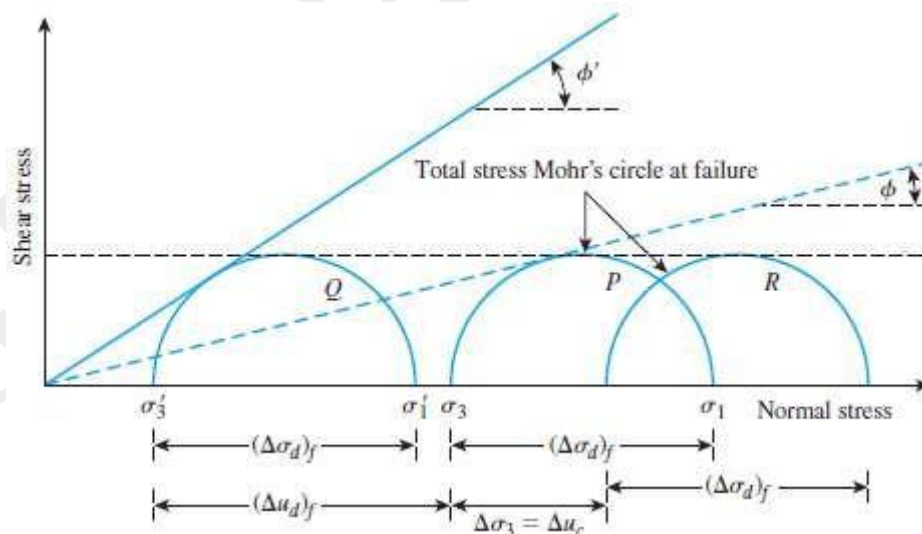


Fig. 11.19: Mohr's circle for total and effective stress and failure envelope  $\phi = 0$

The reason for obtaining the same added axial stress  $\Delta \sigma_{df}$  regardless of the confining pressure can be explained as follows. If clay specimen (No. I) is consolidated at a chamber pressure  $\sigma_3$  and

then sheared to failure without drainage, the total stress conditions at failure can be represented by the Mohr's circle  $P$  in Fig.11.19. The pore pressure developed in the specimen at failure is equal to  $\Delta u_{df}$ . Thus, the major and minor principal effective stresses at failure are, respectively,

$$\sigma'_1 = [\sigma_3 + \Delta\sigma_{df}] - \Delta u_{df} = \sigma_1 - \Delta u_{df} \text{-----} \quad \text{Eq.11.54}$$

$$\sigma'_3 = \sigma_3 - \Delta u_{df} \text{-----} \quad \text{Eq.11.55}$$

$Q$  is the effective stress Mohr's circle drawn with the preceding principal stresses.

Note that the diameters of circles  $P$  and  $Q$  are the same.

Any value of  $\sigma_3$  could have been chosen for testing the specimen. In any case, the deviator stress  $\Delta\sigma_{df}$  to cause failure would have been the same as long as the soil was fully saturated and fully undrained during both stages of the test.

## 11.6 UNCONFINED COMPRESSION TEST

This is a special case of a triaxial compression test; the confining pressure being zero. A cylindrical soil specimen, usually of the same standard size as that for the triaxial compression, is loaded axially by a compressive force until failure takes place. Since the specimen is laterally unconfined, the test is known as 'unconfined compression test'. No rubber membrane is necessary to encase the specimen. The axial or vertical compressive stress is the major principal stress and the other two principal stresses are zero. This test may be conducted on undisturbed or remoulded cohesive soils. It cannot be conducted on coarse-grained soils such as sands and gravels as these cannot stand without lateral support. Also the test is essentially a quick or undrained one because it is assumed that there is no loss of moisture during the test, which is performed fairly fast. Owing to its simplicity, it is often used as a field test, besides being used in the laboratory. The failure plane is not predetermined and failure takes place along the weakest plane. The test specimen is loaded through a calibrated spring by a simple manually operated screw jack at the top of the machine. Different springs with stiffness values ranging from 2 to 20 N/mm may be used to test soils of varying strengths.

The graph of load versus deformation is traced directly on a sheet of paper by means of an autographic recording arm. For any vertical or axial strain, the corrected area can be computed, assuming no change in volume. The axial stress is got by dividing the load by the corrected area. The apparatus is shown in Fig. 11.20.

The specimen is placed between two metal cones attached to two horizontal plates, the upper plate being fixed and the lower one sliding on vertical rods. The spring is supported by a plate and a screw on either side. The plate is capable of being raised by turning a handle so as to apply a compressive load on the soil specimen.

The stress-strain diagram is plotted autographically. The vertical movement of the pen relative to the chart is equal to the extension of the spring, and hence, is proportional to the load. As the lower plate moves upwards, the upper one swings sideways, the weighted arms bearing on a stop. The lateral movement of the pen is thus proportional to the axial strain of the soil specimen. The area of cross-section increases as the specimen gets compressed. A transparent calibrated mask is used to read the stress direct from the chart.

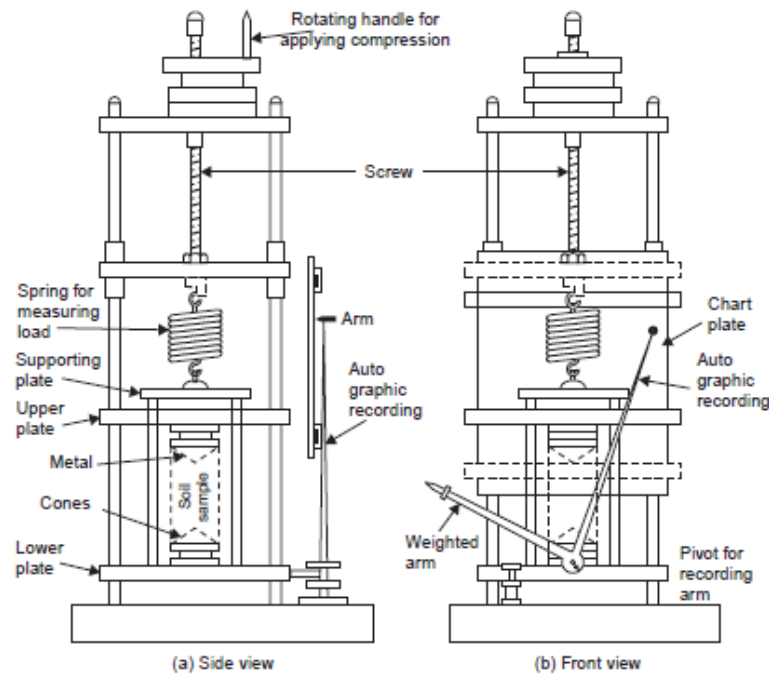


Fig. 11.20: Unconfined Compression Apparatus

Alternatively, a loading frame with proving ring and a dial gauge for measuring the axial compression of the specimen may also be used. The maximum compressive stress is that at the peak of the stress-strain curve. If the peak is not well-defined, an arbitrary strain value such as 20% is taken to represent failure.

### 11.7.1 Mohr's circle for Unconfined Compression Test

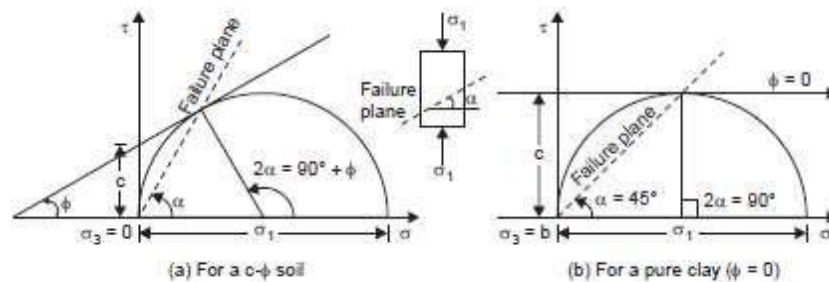


Fig. 11.21: Mohr's circle for Unconfined Compression Test

The Mohr's circles for the unconfined compression test are shown in Fig. 11.21.

$$\sigma_1 = 2c \tan\left(45 + \frac{\phi}{2}\right) \text{-----} \quad \text{Eq.11.56}$$

The two unknowns— $c$  and  $\phi$ —cannot be solved since any number of unconfined compression tests would give only one value for  $\sigma_1$ . Therefore, the unconfined compression test is mostly found useful in the determination of the shearing strength of saturated clays for which  $\phi$  is negligible or zero, under undrained conditions. In such a case, the above equation reduces to

$$\sigma_1 = Q_u = 2c \text{-----} \quad \text{Eq.11.57}$$

Where,  $Q_u$  is the unconfined compression strength.

Thus, the shearing strength or cohesion value for saturated clay from unconfined compression test is taken to be half the unconfined compression strength.

### 11.8 VANE SHEAR TEST

If suitable undisturbed or remoulded samples cannot be got for conducting triaxial or unconfined compression tests, the shear strength is determined by a device called the Shear Vane.

The vane shear test may also be conducted in the laboratory. The laboratory shear vane will be usually smaller in size as compared to the field vane. The shear vane usually consists of four steel plates welded orthogonally to a steel rod, as shown in Fig. 11.22.

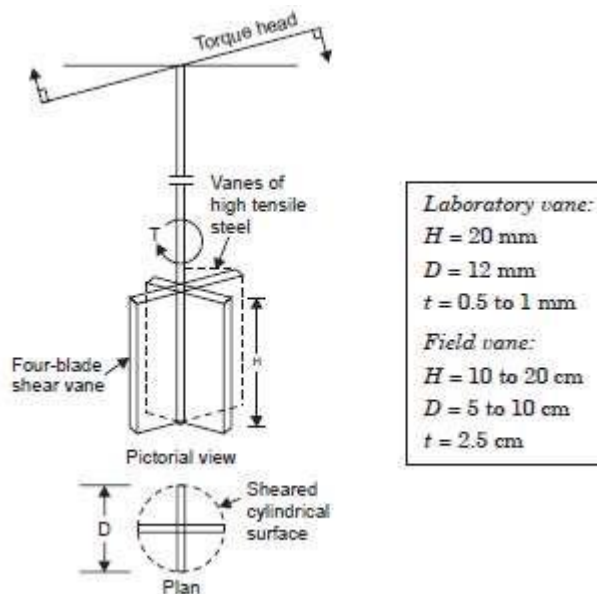


Fig. 11.22: Laboratory shear vane

The applied torque is measured by a calibrated torsion spring, the angle of twist being read on a special gauge. A uniform rotation of about  $1^\circ$  per minute is used. The vane is forced into the soil specimen or into the undisturbed soil at the bottom of a bore-hole in a gentle manner and torque is applied. The torque is computed by multiplying the angle of twist by the spring constant.

The shear strength  $s$  of the clay is given by

$$s = \frac{T}{\pi D \left( \frac{H}{2} + \frac{D}{6} \right)} \quad \text{Eq.11.58}$$

If both the top and bottom of the vane are used in shearing the soil.

Here,  $T$  = torque

$D$  = diameter of the vane

$H$  = height of the vane

If only one end of the vane partakes in shearing the soil, then

$$s = \frac{T}{\pi D \left( \frac{H}{2} + \frac{D}{12} \right)} \quad \text{Eq.11.59}$$

The above Equation may be derived as follows:

The shearing resistance is mobilised at failure along a cylindrical surface of diameter  $D$ , the diameter of the vane, as also at the two circular faces at top and bottom.

The shearing force at the cylindrical surface =  $\pi D.H.s$ ----- Eq.11.60

Where  $s$  is the shearing strength of the soil. The moment of this force about the axis of the vane contributes to the torque and is given by

$$\pi D^2/2.s.h \text{-----} \text{Eq.11.61}$$

For the circular faces at top or bottom, considering the shearing strength of a ring of thickness  $dr$  at a radius  $r$ , the elementary torque is  $(2\pi r dr) \cdot s \cdot r$  and the total for one face is

$$2 \int_0^{\frac{D}{2}} 2\pi sr^2 dr \text{-----} \text{Eq.11.60}$$

The total shear strength developed will be equal to the sum of Eq.11.60 and 61. The maximum moment of the total shear resistance about the axis of torque equals the torque produced  $T$  at failure. Hence

$$T = \pi \frac{D^2}{2} \cdot s \cdot h + 2 \int_0^{\frac{D}{2}} 2\pi sr^2 dr = \pi s \left[ \frac{HD^2}{2} + \frac{D^3}{6} \right] = \pi D^2 s \left[ \frac{H}{2} + \frac{D}{6} \right] \text{-----} \text{Eq.11.61}$$

If only the bottom end partake in the shearing the above equation takes as

$$T = \pi D^2 s \left[ \frac{H}{2} + \frac{D}{12} \right] \text{-----} \text{Eq.11.62}$$

Regarding the shearing stress distribution on the soil cylinder, it is assumed uniform on the cylindrical surface but it is triangular over the shear end faces, varying from zero at the axis of the vane device, to maximum at the edge, as shown in Fig. 11.23.

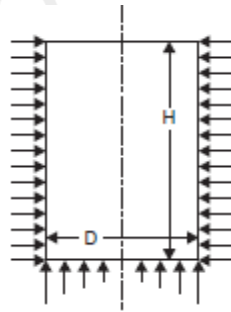


Fig. 11.23: Shearing Distribution on the Sides and Faces of Soil Cylinder in the Vane Shear Test

The vane shear test is particularly suited for soft clays and sensitive clays for which suitable cylindrical specimens cannot be easily prepared.

## 11.9 SHEARING CHARACTERISTICS OF SANDS

The shearing strength in sand may be said to consist of two parts, the internal frictional resistance between grains, which is a combination of rolling and sliding friction and another part known as 'interlocking'. Interlocking, which means locking of one particle by the adjacent ones, resisting movements, contributes a large portion of the shearing strength in dense sands, while it does not occur in loose sands. The Mohr strength theory is not invalidated by the occurrence of

interlocking. The Mohr envelopes merely show large ordinates and steeper slopes for dense soils than for loose ones.

The angle of internal friction is a measure of the resistance of the soil to sliding along a plane. This varies with the density of packing, characterised by density index, particle shape and roughness and particle size distribution. Its value increases with density index, with the angularity and roughness of particles and also with better gradation. This is influenced to some extent by the normal pressure on the plane of shear and also the rate of application of shear.

The 'angle of repose' is the angle to the horizontal at which a heap of dry sand, poured freely from a small height, will stand without support. It is approximately the same as the angle of friction in the loose state.

Some clean sands exhibit slight cohesion under certain conditions of moisture content, owing to capillary tension in the water contained in the voids. Since this is small and may disappear with change in water content, it should not be relied upon for shear strength. On the other hand, even small percentages of silt and clay in sand give it cohesive properties which may be sufficiently large so as to merit consideration.

Unless drainage is deliberately prevented, a shear test on a sand will be a drained one as the high value of permeability makes consolidation and drainage virtually instantaneous. A sand can be tested either in the dry or in the saturated condition. If it is dry, there will be no pore water pressures and if it is saturated, the pore water pressure will be zero due to quick drainage. In either case, the inter granular pressure will be equal to the applied stress. However, there may be certain situations in which significant pore pressures are developed, at least temporarily, in sands. For example, during earth-quakes, heavy blasting and operation of vibratory equipment instantaneous pore pressures are likely to develop due to large shocks or dynamic loads. These may lead to the phenomenon of 'liquefaction' or sudden and total loss of shearing strength, which is a grave situation of lack of stability.

### ***11.9.1 Stress-strain Behaviour of Sands***

The stress-strain behaviour of sands is dependent to a large extent on the initial density of packing, as characterised by the density index. This is represented in Fig. 11.24.

It can be observed from Fig. 11.24 (a), the shear stress (in the case of direct shear tests) or deviator stress (in the case of triaxial compression tests) builds up gradually for an initially loose sand, while for an initially dense sand, it reaches a peak value and decreases at greater values of shear/axial strain to an ultimate value comparable to that for an initially loose specimen. The behaviour of a medium-dense sand is intermediate to that of a loose sand and a dense sand. Intuitively, it should be expected that denser the sand is, the stronger it is. The hatched portion represents the additional strength due to the phenomenon of interlocking in the case of dense sands.

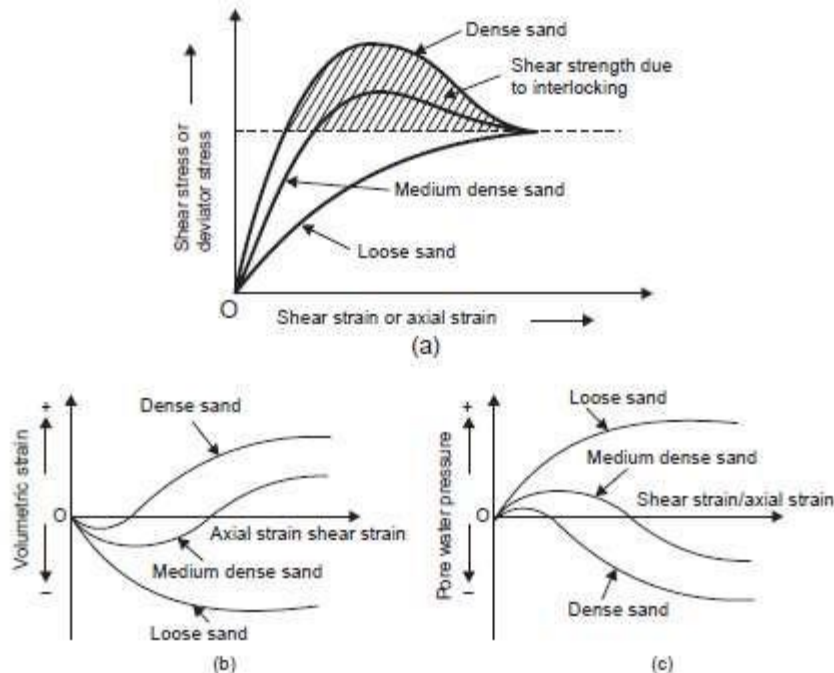


Fig.11.24: Stress-strain Characteristics of Sand

The volume change characteristics of sands are another interesting feature, as depicted in Fig. 11.24(b). An initially dense specimen tends to increase in volume and become loose with increasing values of strain, while an initially loose specimen tends to decrease in volume and become dense. This is explained in terms of the rearrangement of particles during shear.

The changes in pore water pressure during undrained shear, which is rather not very common owing to high permeability of sands, are depicted in Fig. 11.25 (c). Positive pore pressures develop in the case of an initially loose specimen and negative pore pressures develop in the case of an initially dense specimen.

### 11.9.2 Shearing Strength of Sands

The shearing strength of cohesionless soils has been established to depend primarily upon the angle of internal friction which itself is dependent upon a number of factors including the normal pressure on the failure plane. The nature of the results of the shear tests will be influenced by the type of test—direct shear or triaxial compression, by the fact whether the sand is saturated or dry and also by the nature of stresses considered—total or effective.

Each direct shear test is usually conducted under a certain normal stress. Each stress strain diagram therefore reflects the behaviour of a specimen under a particular normal stress. A number of specimens are tested under different normal stresses. It is to be noted that only the effective normal stress is capable of mobilising shear strength. The results when plotted appear as shown in Fig. 11.25.

It may be observed from Fig. 11.25 (a) that the greater the effective normal pressure during shear, the greater is the shearing stress at failure or shearing strength. The shear strength plotted against effective normal pressure gives the Coulomb strength envelope as a straight line, passing



through the origin and inclined at the angle of internal friction to the normal stress axis. It is shown in Fig. 6 (b).

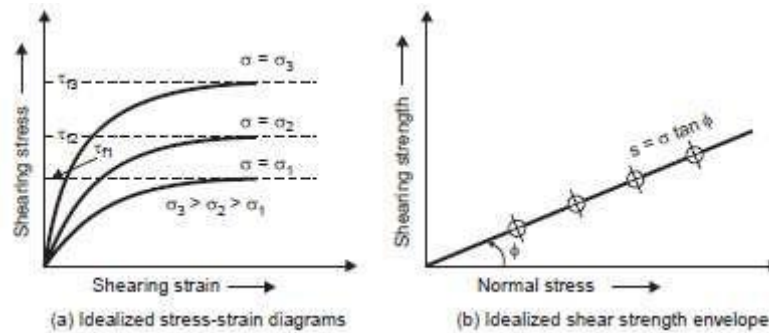


Fig. 11.25: Shear Characteristics of Sand in Direct Test

It may be observed from Fig. 11.25 (a) that the greater the effective normal pressure during shear, the greater is the shearing stress at failure or shearing strength. The shear strength plotted against effective normal pressure gives the Coulomb strength envelope as a straight line, passing through the origin and inclined at the angle of internal friction to the normal stress axis. It is shown in Fig. 11.25 (b). The failure envelope obtained from ultimate shear strength values is assumed to pass through the origin for dry cohesionless soils. The same is true even for saturated sands if the plot is made in terms of effective stresses. In the case of dense sands, the values of  $\phi$  obtained by plotting peak strength values will be somewhat greater than those from ultimate strength values. Ultimate values of  $\phi$  may range from 29 to 35° and peak values from 32 to 45° for sands. The values of  $\phi$  selected for use in practical problems should be related to soil strains expected. If soil deformation is limited, using the peak value for  $\phi$  would be justified. If the deformation is relatively large, ultimate value of  $\phi$  should be used.

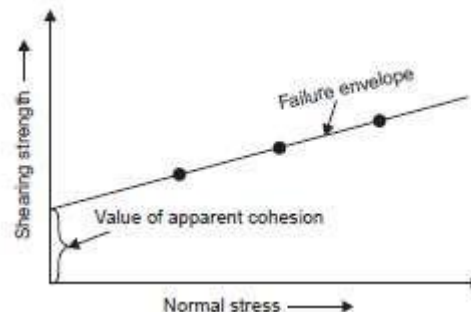


Fig. 11.26: Failure Envelope of sand indicating Apparent Cohesion

If the sand is moist, the failure envelope does not pass through the origin as shown in Fig. 11.26. The intercept on the shear stress axis is referred to as the 'apparent cohesion', attributed to factors such as surface tension of the moisture films on the grains. The extra strength would be lost if the soil were to dry out or to become saturated or submerged. For this reason the extra shear strength attributed to apparent cohesion is neglected in practice.

In the case of triaxial compression tests, different tests with different cell pressure are to be conducted to evaluate the shearing strength and the angle of internal friction. In each test, the



axial normal stress is gradually increased keeping the cell pressure constant, until failure occurs. The value of  $\phi$  is obtained by plotting the Mohr Circles and the corresponding Mohr's envelope. The failure envelope obtained from a series of drained triaxial compression tests on saturated sand specimens initially at the same density index is approximately a straight line passing through the origin, as shown in Fig. 11.27.

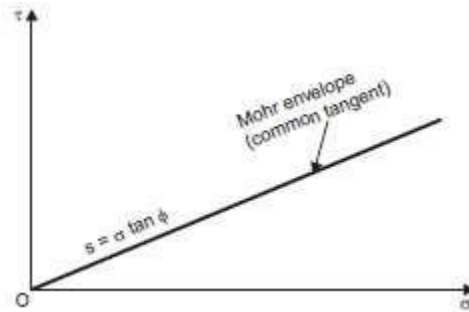


Fig.11.27: Drained Triaxial Test on Saturated Sand

Similar results are obtained when undrained triaxial compression tests are conducted with pore pressure measurements on saturated sand samples and Mohr's circles are plotted in terms of effective stresses. However, if Mohr's circles are plotted in terms of total stresses, the shape of envelopes will be similar to those for a purely cohesive soil. The failure envelope will be approximately horizontal with an intercept on the shearing stress axis, indicating the so called 'apparent cohesion', as shown in Fig. 11.28.

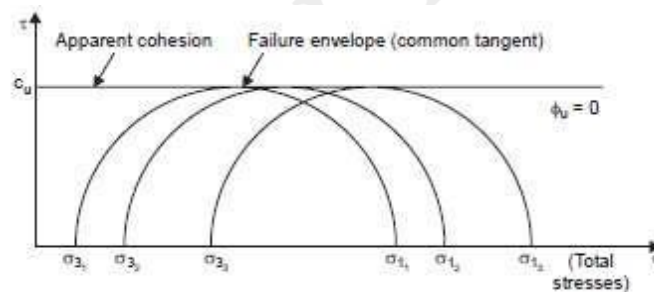


Fig. 11.28: Un-drained Triaxial Test on Saturated Sand (Total Stress)

**Example 1:** The stresses at failure on the failure plane in a cohesionless soil mass were: Shear stress = 4 kN/m<sup>2</sup>; normal stress = 10 kN/m<sup>2</sup>. Determine the resultant stress on the failure plane, the angle of internal friction of the soil and the angle of inclination of the failure plane to the major principal plane.

**Sol:** Resultant stress =  $\sqrt{\sigma^2 + \tau^2}$   
 $= \sqrt{100 + 16} = 10.77 \text{ kN/m}^2$

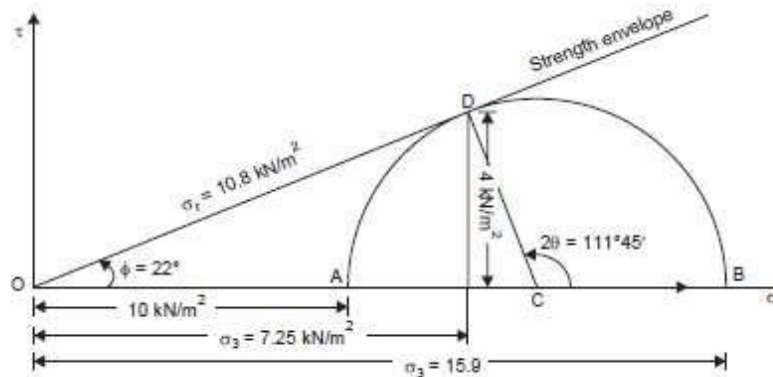
$\tan \phi = 4/10 = 0.4$

$\phi = 21^\circ 48'$

$\theta = 45^\circ + 21^\circ 48' / 2$   
 $= 55^\circ 54'$

**Graphical solution (Fig. 1):**

The procedure is first to draw the  $\sigma$ - and  $\tau$ -axes from an origin  $O$  and then, to a suitable scale, set-off point  $D$  with coordinates (10,4). Joining  $O$  to  $D$ , the strength envelope is got. The Mohr Circle should be tangential to  $OD$  to  $D$ .  $DC$  is drawn perpendicular to  $OD$  to cut  $OX$  in  $C$ , which is the centre of the circle. With  $C$  as the centre and  $CD$  as radius, the circle is completed to cut  $OX$  in  $A$  and  $B$ .

**Fig.1**

By scaling, the resultant stress =  $OD = 10.8 \text{ kN/m}^2$ .

With protractor,  $\phi = 22^\circ$  and  $\theta = 55^\circ 53'$

We also observe that  $\sigma_3 = OA = 7.25 \text{ kN/m}^2$  and  $\sigma_1 = OB = 15.9 \text{ kN/m}^2$ .

**Example 2 :** Calculate the potential shear strength on a horizontal plane at a depth of 3 m below the surface in a formation of cohesionless soil when the water table is at a depth of 3.5 m. The degree of saturation may be taken as 0.5 on the average. Void ratio = 0.50; grain specific gravity = 2.70; angle of internal friction =  $30^\circ$ . What will be the modified value of shear strength if the water table reaches the ground surface ?

$$\begin{aligned} \text{Effective unit weight } \gamma' &= \frac{(G-1)}{(1+e)} \cdot \gamma_w \\ &= \frac{(2.70-1)}{(1+0.5)} \times 10 = 11.33 \text{ kN/m}^3 \end{aligned}$$

Unit weight,  $\gamma$ , at 50% saturation

$$= \frac{(G + S \cdot e)}{(1+e)} \cdot \gamma_w = \frac{(2.70 + 0.5 \times 0.5)}{(1+0.5)} \times 10 = 19.667 \text{ kN/m}^3$$

(a) When the water table is at 3.5 m below the surface:

Normal stress at 3 m depth,  $\sigma = 19.67 \times 3 = 59 \text{ kN/m}^2$

$$\begin{aligned} \text{Shear strength, } s &= \sigma \tan \phi \text{ for a sand} \\ &= 59 \tan 30^\circ = 34 \text{ kN/m}^2 \text{ (nearly).} \end{aligned}$$

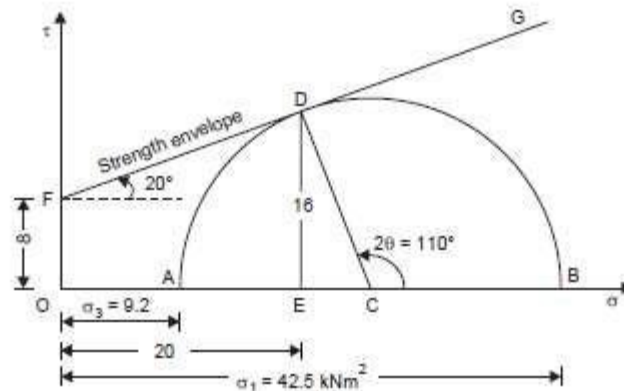
(b) When water table reaches the ground surface:

Effective Normal stress at 3 m depth

$$\bar{\sigma} = \gamma' \cdot h = 11.33 \times 3 = 34 \text{ kN/m}^2$$

$$\begin{aligned} \text{Shear strength, } s &= \bar{\sigma} \tan \phi \\ &= 34 \tan 30^\circ \\ &= 19.6 \text{ kN/m}^2 \text{ (nearly).} \end{aligned}$$

**Example 3 :** The following data were obtained in a direct shear test. Normal pressure = 20 kN/m<sup>2</sup>, tangential pressure = 16 kN/m<sup>2</sup>. Angle of internal friction = 20°, cohesion = 8 kN/m<sup>2</sup>. Represent the data by Mohr's Circle and compute the principal stresses and the direction of the principal planes.



The strength envelope  $FG$  is located since both  $c$  and  $\phi$  are given. Point  $D$  is set-off with co-ordinates (20, 16) with respect to the origin  $O$ ; it should fall on the envelope. (In this case, there appears to be slight discrepancy in the data).  $DC$  is drawn perpendicular to  $FD$  to meet the  $\sigma$ -axis in  $C$ . With  $C$  as centre and  $CD$  as radius, the Mohr's circle is completed. The principal stresses  $\sigma_3$  ( $OA$ ) and  $\sigma_1$  ( $OB$ ) are scaled off and found to be 9.2 kN/m<sup>2</sup> and 42.5 kN/m<sup>2</sup>. Angle  $BCD$  is measured and found to be 110°. Hence the major principal plane is inclined at 55° (clockwise) and the minor principal plane at 35° (counter clockwise) to the plane of shear (horizontal plane, in this case).

**Analytical solution:**

$$\begin{aligned}\sigma_1 &= \sigma_3 N_\phi + 2c \sqrt{N_\phi} \\ N_\phi &= \tan^2 (45^\circ + \phi/2) = \tan^2 55^\circ = 2.04 \\ \sigma_1 &= 2.04 \sigma_3 + 2 \times 8 \times \tan 55^\circ = 2.04 \sigma_3 + 22.88 \quad \dots(1)\end{aligned}$$

$$\begin{aligned}\sigma_n &= \sigma_1 \cos^2 55^\circ + \sigma_3 \sin^2 55^\circ = 20 \\ 0.33 \sigma_1 + 0.67 \sigma_3 &= 20 \quad \dots(2)\end{aligned}$$

Solving,  $\sigma_1 = 42.5$  kN/m<sup>2</sup> and  $\sigma_3 = 9.2$  kN/m<sup>2</sup>, as obtained graphically.

**Example 4 :** A series of shear tests were performed on a soil. Each test was carried out until the sample sheared and the principal stresses for each test were :

Test No.	(kN/m <sup>2</sup> )	(kN/m <sup>2</sup> )
1	200	600
2	300	900
3	400	1200

Plot the Mohr's circles and hence determine the strength envelope and angle of internal friction of the soil.

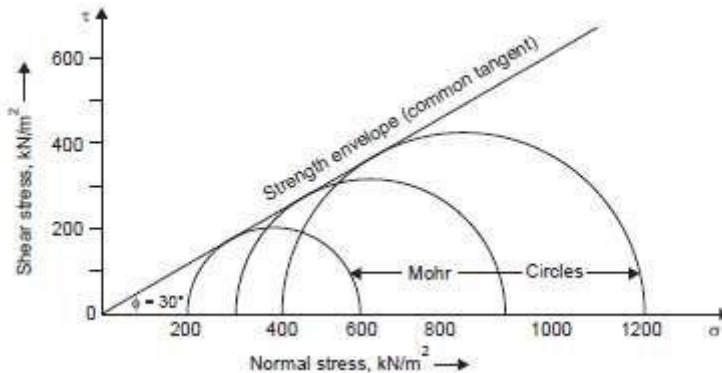
**Sol:**

The data indicate that the tests are triaxial compression tests; the Mohr's circles are plotted with  $(\sigma_1 - \sigma_3)$  as diameter and the strength envelope is obtained as the common tangent.

The angle of internal friction of found to be 30°, by measurement with a protractor from

**Example 5 :** A particular soil failed under a major principal stress of  $300 \text{ kN/m}^2$  with a corresponding minor principal stress of  $100 \text{ kN/m}^2$ . If, for the same soil, the minor principal stress had been  $200 \text{ kN/m}^2$ , determine what the major principal stress would have been if

- (a)  $\phi = 30^\circ$  and (b)  $\phi = 0^\circ$ .



The Mohr circle of stress is drawn to which the strength envelope will be tangential; the envelopes for  $\phi = 0^\circ$  and  $\phi = 30^\circ$  are drawn. Two stress circles, each starting at a minor principal stress value of  $200 \text{ kN/m}^2$ , one tangential to  $\phi = 0^\circ$  envelope, and the other tangential to  $\phi = 30^\circ$  envelope are drawn.

The corresponding major principal stresses are scaled off as  $400 \text{ kN/m}^2$  and  $600 \text{ kN/m}^2$ .

**Analytical solution:**

- (a)  $\phi = 30^\circ$ ;

$$\sigma_3 = 100 \text{ kN/m}^2, \sigma_1 = 300 \text{ kN/m}^2$$

$$\frac{\sigma_2}{\sigma_1} = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = 1/3$$

The given stress circle will be tangential to the strength envelope with  $\phi = 30^\circ$ .

With  $\sigma_3 = 200 \text{ kN/m}^2$ ,  $\sigma_1 = 3 \times 200 = 600 \text{ kN/m}^2$ ,

if the circle is to be tangential to the strength envelope  $\phi = 30^\circ$  passing through the origin.

- (b)  $\phi = 0^\circ$ ;

If the given stress circle has to be tangential to the strength envelope  $\phi = 0^\circ$ , the envelope has to be drawn with  $c = \tau = 100 \text{ kN/m}^2$ . The deviator stress will then be  $200 \text{ kN/m}^2$ , irrespective of the minor principal stress.

**Example 6** In a triaxial shear test conducted on a soil sample having a cohesion of  $12 \text{ kN/m}^2$  and angle of shearing resistance of  $36^\circ$ , the cell pressure was  $200 \text{ kN/m}^2$ . Determine the value of the deviator stress at failure.

**Sol:**

Hence  $\sigma_1 = 200 + 200 = 400 \text{ kN/m}^2$  for  $\sigma_3 = 200 \text{ kN/m}^2$ .

**Analytical solution:**

$$c = 12 \text{ kN/m}^2$$

$$\phi = 36^\circ$$

$$\sigma_3 = 200 \text{ kN/m}^2$$

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

where  $N_\phi = \tan^2 (45^\circ + \phi/2)$ .

$$N_\phi = \tan^2 (45^\circ + 18^\circ) = \tan^2 63^\circ = 3.8518$$

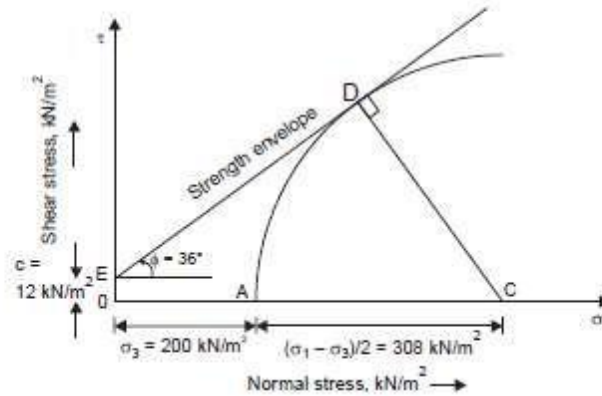
$$\sqrt{N_\phi} = \tan 63^\circ = 1.9626$$

$$\therefore \sigma_1 = 200 \times 3.8518 + 2 \times 12 \times 1.9626 = 817.5 \text{ kN/m}^2$$

$$\text{Deviator stress} = \sigma_1 - \sigma_3 = (817.5 - 200) \text{ kN/m}^2 = 617.5 \text{ kN/m}^2$$

Graphical Sol:

The strength envelope is drawn through  $E$  on the  $\tau$ -axis,  $OE$  being equal to  $C = 12 \text{ kN/m}^2$  to a convenient scale, at an angle  $\phi = 36^\circ$  with the  $\sigma$ -axis. The cell pressure,  $\sigma_3 = 200 \text{ kN/m}^2$  is plotted as  $OA$ . With centre on the  $\sigma$ -axis, a circle is drawn to pass through  $A$  and be tangential to the envelope, by trial and error.  $AC$  is scaled-off,  $C$  being the centre of the Mohr's circle, which is  $(\sigma_1 - \sigma_3)/2$ . The deviator stress is double this value. In this case the result is  $616 \text{ kN/m}^2$ . (Fig. 8.54).



**Example 7 :** The shearing resistance of a soil is determined by the equation  $s = c' + \sigma' \tan \phi'$ . Two drained triaxial tests are performed on the material. In the first test the all-round pressure is  $200 \text{ kN/m}^2$  and failure occurs at an added axial stress of  $600 \text{ kN/m}^2$ . In the second test all-round pressure is  $350 \text{ kN/m}^2$  and failure occurs at an added axial stress of  $1050 \text{ kN/m}^2$ . What values of  $c'$  and  $\phi'$  correspond to these results?

Sol:

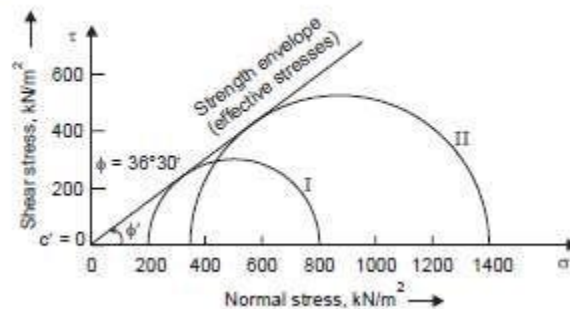
**Analytical method:**

Since the tests are drained tests, we may assume  $c' = 0$ . On this basis, we may obtain  $N_\phi$ .

From both tests,  $N_\phi = \sigma_1/\sigma_3 = 4$

$$\begin{aligned} \therefore \sqrt{N_\phi} &= \tan(45^\circ + \phi'/2) = 2 \\ \text{or } 45^\circ + \phi'/2 &= 63^\circ 25', \phi'/2 = 18^\circ 25' \\ \therefore \phi' &= 36^\circ 50'. \end{aligned}$$

**Graphical method:**





## 12.0 SOIL STABILIZATION

### INTRODUCTION

‘Soil Stabilisation’, in the broadest sense, refers to the procedures employed with a view to altering one or more properties of a soil so as to improve its engineering performance. Soil Stabilisation is only one of several techniques available to the geotechnical engineer and its choice for any situation should be made only after a comparison with other techniques indicates it to be the best solution to the problem.

It is a well known fact that, every structure must rest upon soil or be made of soil. It would be ideal to find a soil at a particular site to be satisfactory for the intended use as it exists in nature, but unfortunately, such a thing is of rare occurrence.

The alternatives available to a geotechnical engineer, when an unsatisfactory soil is met with, are (i) to bypass the bad soil (*e.g.*, use of piles), (ii) to remove bad soil and replace with good one (*e.g.*, removal of peat at a site and replacement with selected material), (iii) redesign the structure (*e.g.*, floating foundation on a compressible layer), and (iv) to treat the soil to improve its properties.

The last alternative is termed soil stabilisation. Although certain techniques of stabilisation are of a relatively recent origin, the art itself is very old. The original objective of soil stabilisation, was, as the name implies, to increase the strength or stability of soil. However, techniques have now been developed to alter almost every engineering property of soil. The primary aim may be to alter the strength and/or to reduce its sensitivity to moisture changes. The most common application of soil stabilisation is the strengthening of the soil components of highway and airfield pavements.

### 12.1 CLASSIFICATION OF THE METHODS OF STABILISATION

A completely consistent classification of soil stabilisation techniques is difficult. Classifications may be based on the treatment given to soil, on additives used, or on the process involved. Broadly speaking, soil stabilisation procedures may be brought under the following two heads:

#### I. *Stabilisation without additives*

#### II. *Stabilisation with additives*

Stabilisation without additives may be ‘mechanical’—rearrangement of particles through compaction or addition or removal of soil particles. It may be by ‘drainage’—drainage may be achieved by the addition of external load, by pumping, by electro—osmosis, or by application of a thermal gradient—heating or cooling.

Stabilisation with additives may be cement stabilisation (that is, soil cement), bitumen stabilisation, or chemical stabilisation (with fly ash, lime, calcium or sodium chloride, sodium silicate, dispersants, physico-chemical alteration involving ion-exchange in clay-minerals or injection stabilisation by grouting with soil, cement or chemicals).

The appropriate method for a given situation must be chosen by the geotechnical engineer based on his experience and knowledge. Comparative laboratory tests followed by limited field tests, should be used to select the most economical method that will serve the particular problem on hand. Field-performance data may help in solving similar problems which arise in future. It must be remembered, however, that soil stabilisation is not always the best solution to a problem.

### **Mechanical Stabilisation**

‘Mechanical stabilisation’ means improving the soil properties by rearrangement of particles and densification by compaction, or by changing the gradation through addition or removal of soil particles.

#### ***Rearrangement of particles—compaction***

The process of densification of a soil or ‘compaction’, as it is called, is the oldest and most important method. In addition to being used alone, compaction constitutes an essential part of a number of other methods of soil stabilisation.

The important variables involved in compaction are the moisture content, compactive effort or energy and the type of compaction. The most desirable combination of the placement variables depends upon the nature of the soil and the desired properties. Fine-grained soils are more sensitive to placement conditions than coarse-grained soils.

Compaction has been shown to affect soil structure, permeability, compressibility characteristics and strength of soil and stress-strain characteristics (Leonards, 1962).

#### ***Change of gradation—addition or removal of soil particles***

The engineering behaviour of a soil depends upon (among other things) the grain-size distribution and the composition of the particles. The properties may be significantly altered by adding soil of some selected grain-sizes, and, or by removing some selected fraction of the soil. In other words, this approach consists in manipulating the soil fractions to obtain a suitable grading, which involves mixing coarse material or gravel (called ‘aggregate’), sand, silt and clay in proper proportions so that the mixture when compacted attains maximum density and strength. It may involve blending of two or more naturally available soils in suitable proportions to achieve the desired engineering properties for the mixture after necessary compaction.

Soil materials can be divided into two fractions, the granular fraction or the ‘aggregate’, retained on a 75-micron I.S. Sieve, and the fine soil fraction or the ‘binder’, passing this sieve. The aggregate provides strength by internal friction and hardness or incompressibility, while the binder provides cohesion or binding property, water-retention capacity or imperviousness and also acts as a filler for the voids of the aggregate.

The relative amounts of aggregate and binder determine the physical properties of the compacted stabilised soil. The optimum amount of binder is reached when the compacted binder fills the voids without destroying all the grain-to-grain contacts of coarse particles. Increase in the binder beyond this limit results in a reduction of internal friction, a slight increase in cohesion and greater compressibility. Determination of the optimum amount of binder is an important component of the design of the mechanically stabilised mixture.

Mechanical stabilisation of this type has been largely used in the construction of low cost roads. Guide specifications have been developed based on past experience, separately for base courses and surface courses.

The grading obtained by a simple rule given by Fuller has been found to be satisfactory:

$$\text{Percent passing a particular sieve} = \sqrt{\frac{d}{D}} 100$$

Where,  $d$  = aperture size of the sieve, and

$D$  = size of the largest particle.

Suggested gradings for mechanically stabilised base and surface courses for roads are given in Table 1.

I.S. Sieve size	Per cent passing				Base or surface course	
	Base course Max. size		Surface course Max. size		Max. size	
	80 mm	40 mm	20 mm	20 mm	10 mm	5 mm
80 mm	100	—	—	—	—	—
40 mm	80–100	100	—	—	—	—
20 mm	60–80	80–100	100	100	—	—
10 mm	45–65	55–80	80–100	80–100	100	—
5 mm	30–50	40–60	50–75	60–85	80–100	100
2.36 mm	—	30–50	35–60	45–70	50–80	80–100
1.18 mm	—	—	—	35–60	40–65	50–80
600 micron	10–30	15–30	15–35	—	—	30–60
300 micron	—	—	—	20–40	20–40	20–45
75 micron	5–15	5–15	5–15	10–25	10–25	10–25

If the primary aim is to reduce the permeability of a soil, sodium montmorillonite—a clay mineral, called “bentonite”—may be added. For example, the permeability of a silty sand could be reduced from 10–4 cm/s to 10–9 cm/s by the addition of 10% of bentonite. However, it must be remembered that bentonite is costly and its effectiveness may be reduced by flowing water, and wetting and drying. Naturally available local clay can be blended with pervious soils to result in a more nearly permanent blanket; this may be a much cheaper and superior approach, if such a material is available in the proximity of the site

**Note:**

1 Not less than 10% should be retained between each pair of successive sieves specified, excepting the largest pair.

2. Material passing I.S. Sieve No. 36 shall have the following properties:

*For base courses: For surface courses:*

Liquid limit  $>|$  25% Liquid limit  $>|$  35%

Plasticity Index  $>|$  6% Plasticity Index : between 4 and 9.



Gravel is used for base courses of pavements and for filter courses. The presence of fines to an extent more than the optimum might make the gravel unsatisfactory. The limit for the fines may be 3 to 7%, depending upon its intended use. The upper limit is for filter purposes.

An obvious treatment, for a gravel with larger amount of fines than desired, is to wash out excess fines. This may sound very easy, but it is not that simple in practice. When supplies of dirty gravel along with a satisfactory source of water are available locally, the procedure of removal of fines from gravel by washing is employed.

### **Cement Stabilisation**

Portland cement is one of the most widely used additives for soil stabilisation. A mixture of soil and cement is called “soil-cement”. If a small percentage of cement is added primarily to reduce the plasticity of fat soils, the mixture is said to be a “cement-modified soil”. If the soil cement as enough water which facilitates pouring it as mortar, it is said to be a ‘plastic soil cement’. It is used in canal linings.

The chemical reactions of cement with the silicious soil in the presence of water are believed to be responsible for the cementing action. Many of the grains of the coarse fraction get cemented together, but the proportion of clay particles cemented is small.

Almost any inorganic soil can be successfully stabilised with cement; organic matter may interfere with the cement hydration.

Soil-cement has been widely used for low-cost pavements for highways and airfields, and as bases for heavy traffic. Generally, it is not recommended as a wearing coarse in view of its low resistance to abrasion.

### ***Factors affecting soil-cement***

The important factors which affect the properties of soil-cement are the nature of the soil, cement content, compaction, and the method of mixing.

### ***Nature of the soil***

Almost all soils, devoid of organic matter and capable of being pulverised, can be stabilized with the addition of cement. The requirement of cement will increase with the increase in specific surface of the soil; in other words, it increases with the fines content. Expansive clays are difficult to deal with. Well-graded soils with less than 50% of particles finer than 75- $\mu$  and a plasticity index less than 20 are most suitable for this method of stabilisation. Approximate limits of gradation of soil for economic stabilisation with cement are obtained by research (HRB, 1943).

Soils containing more than 2% of organic matter are generally considered to be unsuitable, since the strength of soil-cement is reduced by the organic matter interfering with the hydration of cement. The presence of sulphates also renders a soil unsuitable for stabilization with cement.

The nature of the exchangeable ions on the soil grains is an important factor. Calcium is the most desirable ion for the case of cement stabilisation. Addition of less than 1% of lime or calcium chloride may render a soil more suitable for stabilisation with cement in spite of the presence of organic matter.

### ***Cement content***

The normal range of cement content used is 5 to 15% by weight of the dry soil, finer soils requiring greater quantity of cement. The more the cement content, the greater is the strength of the resulting soil-cement. A compressive strength of 2000 to 3000 kN/m<sup>2</sup> as obtained from a test on a cylinder of soil-cement after 7 days of curing must be satisfactory. High early strength cement yields better results than ordinary cement.

### ***Compaction***

Adequate compaction is essential. Optimum moisture is to be used in the process as there is no problem of stability as for concrete.

### ***Mixing***

Uniform mixing will lead to strong soil-cement. The efficiency of mixing depends upon the type of plant used. Mixing should not be done after hydration has begun.

### ***Admixtures***

Addition of about 0.5 to 1.0% of certain chemicals such as lime or calcium chloride to soil cement has been found to accelerate the set and to improve the properties of the final products (Lambe, Michaels and Mohr, 1959).

### ***Designing and testing soil-cement***

Soil-cement mix design consists of selecting the amount of cement, the amount of water and the compaction density to be achieved in the field. The thickness of the stabilised soil and also the placement conditions are to be decided upon. The thickness of a soil-cement base is to be taken equal to that required for a granular base for a good sub-grade, and as equal to 75% of that required for a granular base for a poor sub-grade. Usually 15 to 20 cm thickness is adequate for a soil-cement base.

The procedures adopted for design are:

- (i) Complete method, using moisture-density, freeze-thaw, and wet-dry tests;
- (ii) Short-cut method for sandy soils, using moisture-density and strength tests in conjunction with the charts prepared by the Portland Association; and
- (iii) Rapid method, using moisture-density tests and visual inspection.

### ***Field construction of soil-cement***

The steps in the construction of soil-cement are:

- (i) Pulverization of soil
- (ii) Adding water and cement
- (iii) Mixing
- (iv) Spreading and compaction
- (v) Finishing and curing

In advanced countries most of the operations are mechanised. The method may be mixed-in place, travelling plant, or stationary plant type.

### ***Applications***

Soil-cement is used as base course for pavements for light traffic. Pressed soil-cement blocks can be used in place of bricks. Rammed earth walls with just 2% cement are used for low-cost

housing. Soil-cement blocks may be used in place of concrete blocks for pitching of banks of canals or canal linings.

### **Bitumen Stabilisation**

Bituminous materials such as asphalts and tars have been used for soil stabilisation. This method is better suited to granular soils and dry climates.

‘Bitumens’ are nonaqueous system of hydrocarbons which are completely soluble in ‘Carbon disulphide’. ‘Asphalts’ are natural materials or refined petroleum products, which are bitumens. ‘Tars’ are bituminous condensates produced by the destructive distillation of organic materials such as coal, oil, lignite and wood. (Lambe, 1962).

Most bitumen stabilisation has been with asphalt. Asphalt is usually too viscous to be incorporated directly with soil. Hence, it is either heated or emulsified or cut back with a solvent like gasoline, to make it adequately fluid.

Tars are not emulsified but are heated or cut back prior to application. Soil-asphalt is used mostly for base courses of roads with light traffic.

Bitumen stabilises soil by one or both of two mechanisms: (i) binding soil particles together, and (ii) making the soil water-proof and thus protecting it from the deleterious effects of water. Obviously, the first mechanism occurs in cohesionless soils, and the second in cohesive soils, which are sensitive to water. Asphalt coats the surfaces of soil particles and protects them from water. It also plugs the voids in the soil, inhibiting a flow of pore water.

Bitumen stabilisation may produce one of the following:

- (i) Sand-bitumen
- (ii) Soil-bitumen
- (iii) Water-proof mechanical stabilisation
- (iv) Oiled earth

#### ***Sand-bitumen***

Cohesionless soils like sand stabilised by bitumen are called sand-bitumen. The primary function of bitumen is to bind the sand grains. The quantity of asphalt may range from 5 to 12%.

#### ***Soil-bitumen***

Cohesive soil stabilised by bitumen is referred to as soil-bitumen; the primary objective is to make it water-proof and preserve its cohesive strength.

For best results the soil must conform to the following requirements:

Max. size : less than one-third the compacted thickness

Passing 4.75 mm sieve: greater than 50%

Passing 425- $\mu$  Sieve: 35 to 100%

Passing 75- $\mu$  sieve : 10 to 30%

Liquid limit: less than 40%

Plasticity Index: less than 18

#### ***Water-proof mechanical stabilisation***

Small quantities of bitumen—1 to 3%—may be added to mechanically stabilised soils to make them water-proof.

***Oiled earth***

Slow-curing and medium-curing road oils are sprayed to make the earth water-resistant and resistant to abrasion. The oils penetrate a short depth into the soil without any mechanical mixing.

***Admixtures***

The addition of small quantities of phosphorus pentoxide or certain amines was found to improve the effectiveness of asphalt as a soil stabiliser.

The construction of soil-asphalt is very much similar to that of soil-cement; usual thickness ranges from 15 to 20 cm as with soil-cement.

**Chemical Stabilisation**

Chemical stabilisation refers to that in which the primary additive is a chemical. The use of chemicals as secondary additives to increase the effectiveness of cement and of asphalt has been mentioned earlier.

Lime and salt have found wide use in the field. Some chemicals are used for stabilizing the moisture in the soil and some for cementation of particles. Certain aggregates and dispersants have also been used.

***Lime stabilisation***

Lime is produced from natural limestone. The hydrated limes, called 'slaked lines', are the commonly used form for stabilisation.

In addition to being used alone, lime is also used in the following admixtures, for soil stabilisation:

- (i) Lime-fly ash (4 to 8% of hydrated lime and 8 to 20% of fly-ash)
- (ii) Lime-portland cement
- (iii) Lime-bitumen

The use of lime as a soil stabiliser dates back to Romans, who used it in the construction of the 'Appian way' in Rome. This road has given excellent service and is maintained as a traffic artery even today.

There are two types of chemical reactions that occur when lime is added to wet soil. The first is the alteration of the nature of the adsorbed layer through ion exchange of calcium for the ion naturally carried by the soil, or a change in the double layer on the soil colloids. The second is the cementing action or pozzolanic action which requires a much longer time. This is considered to be a reaction between the calcium with the available reactive alumina or silica from the soil.

***Lime has the following effects on soil properties:***

Lime generally increases the plasticity index of low-plasticity soil and decreases that of highly plastic soils; in the latter case, lime tends to make the soil friable and more easily handled in the field.

It increases the optimum moisture content and decreases the maximum compacted density; however, there will be an increase in strength. About 2 to 8% of lime may be required for coarse-grained soils, and 5 to 10% for cohesive soils.

Certain sodium compounds (*e.g.*, sodium hydroxide and sodium sulphate), as secondary additives, improve the strength of soil stabilised with lime.

Lime may be applied in the dry or as a slurry. Better penetration is obtained when it is used as a slurry. The construction of lime-stabilised soil is very much similar to that of soilcement. The important difference is that, in this case, no time limitation may be placed on the operations, since the lime-soil reactions are slow. Care should be taken, however, to prevent the carbonation of lime. Lime stabilisation has been used for bases of pavements.

### ***Salt stabilisation***

Calcium chloride and sodium chloride have been used for soil stabilisation. Calcium chloride is hygroscopic and deliquescent. It absorbs moisture from the atmosphere and retains it. It also acts as a soil flocculant. The action of sodium chloride is similar.

The effect of salt on soil arises from colloidal reactions and the alteration of the characteristics of soil water. Salt lowers the vapour pressure of pore water and also the freezing point; the frost heave will be reduced because of the latter phenomenon.

The main disadvantage is that the beneficial effects of salt are lost, if the soil gets leached.

### ***Lignin and chrome-lignin stabilisation***

Lignin is one of the major constituents of wood and is obtained as a by-product during the manufacture of paper from wood. Lignin, both in powder form and in the form of sulphite liquor, has been used as an additive to soil for many years. A concentrated solution, partly neutralised with calcium base, known as Lignosol, has also been used.

The stabilising effects of lignin are not permanent since it is soluble in water; hence periodic applications may be required. In an attempt to improve the action of lignin, the ‘Chromelignin process’ was developed (Smith, 1952). The addition of sodium bi-chromate or potassium bi-chromate to the sulphite waste resulted in the formation of an insoluble gel.

If the lignin is not neutralised, it is acid and acts as a soil aggregant; when neutralized as with Lignisol, it acts as a dispersant. Chrome lignin imparts considerable strength to soils as a cementing agent (Lambe, 1962).

### ***Stabilisers with water-proofers***

It is well known that cohesive soils possess considerable strength when they are dry. When they have access to water, they imbibe it and lose strength. Water-proofers, *i.e.*, chemicals which prevent the deleterious effects of water on soils, are useful in such cases. Siliconates, amines and quaternary ammonium salts fall in this category.

Water-proofers do not increase the strength, but help the soil retain its strength even in the presence of water.

### ***Stabilisation with natural and synthetic resins***

Certain natural as well as synthetic resins, which are obtained by polymerisation of organic monomers, have also been used for soil stabilisation. They act primarily as water-proofers. Vinol resin and Rosin, both of which are obtained from pine trees, are the commonly used natural resins. Aniline-furfural, polyvinyl alcohol (PVA), and calcium acrylate are commonly used

synthetic resins. Asphalt and lignin, which are also resinous materials, have already been discussed separately.

### ***Aggregants and dispersants***

Aggregants and dispersants are chemicals which bring about modest changes in the properties of soil containing fine grains. These materials function by altering the electrical forces between the soil particles of colloidal size, but provide no cementing action. They affect the plasticity, permeability and strength of the soil treated. Low treatment levels are adequate for the purpose. Aggregants increase the net electrical attraction between adjacent fine-grained soil particles and tend to flocculate the soil mass. Inorganic salts such as calcium chloride and ferric chloride, and polymers such as Krilium are important examples. Change in adsorbed water layers, ion-exchange phenomena and increase in ion concentration are the possible mechanisms by which the aggregants work.

Dispersants are chemicals which increase the electrical repulsion between adjacent fine grained soil particles, reduce the cohesion between them, and tend to cause them to disperse.

Phosphates, sulphonates and versanates are the most common dispersants, which tend to decrease the permeability. Ion exchange and anion adsorption are the possible mechanisms by which the dispersants work.

### ***Miscellaneous chemical stabilisers***

Sodium silicate can be used as a primary stabiliser as well as a secondary additive to conventional stabilisers such as cement. Injection is the usual process by which this is used. Phosphoric acid also has been used to some extent.

Molasses, tung oil, sodium carbonate, paraffin and hydrofluoric acid are some miscellaneous chemicals which have been considered but have not received any extensive application.

### ***Injection Stabilisation***

Injection of the stabilising agent into the soil is called 'Grouting'. This process makes it possible to improve the properties of natural soil and rock formations, without excavation, processing, and recompaction. Grouting may have one of the two objectives—to improve strength properties or to reduce permeability. This is achieved by filling cracks, fissures and cavities in the rock and the voids in soil with a stabiliser this is initially in a liquid state or in suspension and which subsequently solidifies or precipitates.

Injection is a very common technique in the oil industry; petroleum engineers frequently use this method for sealing or operating wells. Injection techniques, unfortunately, are rather complex. The selection of proper grout material and appropriate technique can normally be made best only after field exploration and testing. The results of the injection process are rather difficult to assess.

*Grouting must be called an art rather than a science.*

### ***Grouting materials***

The following have been used for grouting:

- (i) Soil
- (ii) Portland cement
- (iii) Bitumen

## (iv) Chemicals

Bitumen is not used as much as other materials for this purpose.

The properties of the grout must fit the soil or rock formation being injected. The dimensions of the pores or fissures determine the size of grout particle so that these can penetrate

$$D_{85}(\text{Grout}) < D_{15}(\text{Soil})$$

$$D_{85}(\text{Grout}) < \frac{1}{3} B(\text{Fissure})$$

$$R_g = \frac{D_{85}(\text{Grout})}{D_{15}(\text{Soil})} > 15$$

Where  $R_g$  = Grout ability ratio

Viscosity and rate of hardening are important characteristics of the grout material. Low viscosity and slow hardening permits penetration to thin fissures and small voids whereas high viscosity and rapid hardening restrict flow to large voids.

The grout must not be unduly diluted or washed away by ground water. Insoluble or rapid setting grouts are used in situations where there is ground water flow.

‘Mudjacking’ is a form of soil injection used to raise highway pavements, railway tracks and even storage tanks. This technique consists of injecting a mixture of soil, Portland cement and water to shallow depths at relatively low pressures.

Cement grouting has been used to stabilise rock formations, as also alluvial sands and gravels. A number of chemicals have been used for grouting; among them, sodium silicate in water, known as ‘water glass’ is the most common. This solution contains both free sodium hydroxide and colloidal silicic acid. The addition of certain salts such as calcium chloride, magnesium chloride, ferric chloride and magnesium sulphate, or of certain acids such as hydrochloric acid and sulphuric acid, results in the formation of an insoluble silica gel.

On ageing, the gel shrinks and cracks. Hence, the effectiveness of silicate injection in the presence of ground water remains doubtful.

The grouting plant includes the material handling system, mixers, pumps and delivery hoses. The mixing of components is done by a proportioning valve or pump at the point of injection. A perforated pipe is driven into soil to the level of grouting; if it is rock, grouting holes are drilled. The injection pattern depends on the purpose of grouting. Generally a grid pattern is used. The spacing may be 6 to 15 m. sufficient pressure is used to force the grout into voids and fissures.

### Thermal Stabilization

Thermal change causes a marked improvement in the properties of the soil. Thermal stabilization is done either by heating the soil or by cooling it. Heating: As the soil is heated, its water content decreases. Electric repulsion between clay particles is decreased and the strength of the soil is increased.

Freezing: cooling causes small loss of strength of clayey soils due to an increase in inter particles repulsion. However, if the temperature is reduced to the freezing point, the pore water freezes and the soil is stabilized.

### Electrical Stabilization

Electrical stabilization of clayey soils is done by a process known as electro-osmosis. As a direct current (DC) is passed through a clayey soil, pore water migrates to the negative electrode (cathode). It occurs because of attraction of positive ions (cations) that are present in water towards cathode. The strength of the soil is considerably increased due to removal of water. Electro-osmosis is an expensive method, and is mainly used for drainage of cohesive soils. Incidentally, the properties of the soil are also improved.

#### **Stabilization by Geo-textile and Fabrics**

Geotextiles are porous fabrics made of synthetic materials such as polyethylene, polyester, nylons and polyvinyl chloride. Woven, non-woven and grid form varieties of geotextiles are available. Geotextiles have a high strength.

When properly embedded in soil, it contributes to its stability. It is used in the construction of unpaved roads over soft soils. Reinforcing the soil for stabilization by metallic strips into it and providing an anchor or tie back to restrain a facing skin element. Past research has shown that the strength and load-bearing capacity of subgrades and base course materials can be improved through the inclusion of non biodegradable reinforcing materials, such as fibers, geotextiles, geogrids, and geo-composites. Use of these materials can improve the performance and durability of future highways and may reduce the cost of construction. At present, most of the research on these materials is based on tests conducted in the laboratory that are only partially complete. Further laboratory tests and evaluations will be necessary to develop design specifications based on material properties, and these specifications will need to be verified using large-scale field tests.

#### **Recycled and Waste Products**

Improved chemical and mechanical stabilization techniques are needed for such waste materials as crushed old asphalt pavement, copper and zinc slag, paper mill sludge, and rubber tire chips. The need to recycle many potentially hazardous materials, it will be necessary to develop a realistic, economical and effective means of assessing the risk of pollution posed by these materials through leachates and emissions. In some cases, risk evaluation is hampered by restrictive environmental constraints, and this issue needs to be addressed as well.

#### **Rice Husk ash Stabilization**

Disposal of solid waste on the land fill can be minimized if the waste is having desirable properties such that they can be utilized for various geotechnical application viz. land reclamation, construction of embankment etc. There are several methods used for improving geotechnical properties of problematic soils that includes densification (such as shallow compaction, dynamic deep compaction, pre-loading), drainage, inclusions (such as geosynthetics and stone columns), and stabilizations. Chemical stabilization of the problematic soils is especially significant in concerning with the treatment of soft fine-grained, expansive soils, and collapsible loess deposits. Soil stabilization is the process which is used to improve the engineering properties of the soil and thus making it more stable. Soil stabilization is required when the soil available for construction is not suitable for the intended purpose. It includes compaction, pre-consolidation, drainage and many other such processes. Rice husk ash (RHA) is



a pozzolanic material that could be potentially used in soil stabilization, though it is moderately produced and readily available. When rice husk is burnt under controlled temperature, ash is produced and about 17%-25% of rice husk's weight remains ash. Rice husk ash and rice straw and barks are rich in silica and make an excellent pozzolana. Pozzolanas are siliceous and aluminous materials, which in itself possess little or no cementations value, but will, in finely divided form and in the presence of moisture, chemically react with calcium hydroxide at ordinary temperature to form compounds possessing cementations properties. The Rice Husk Ash would appear to be an inert material with the silica in the crystalline form suggested by the structure of the particles, it is very unlikely that it would react with lime to form calcium silicates. It is also unlikely that it would be as reactive as fly ash, which is more finely divided. So Rice Husk Ash would give great results when it used as a stabilizing material. The ash would appear to be a very suitable light weight fill and should not present great.